Multiscale Geometric and Spectral Analysis of Plane Arrangements

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Abstract

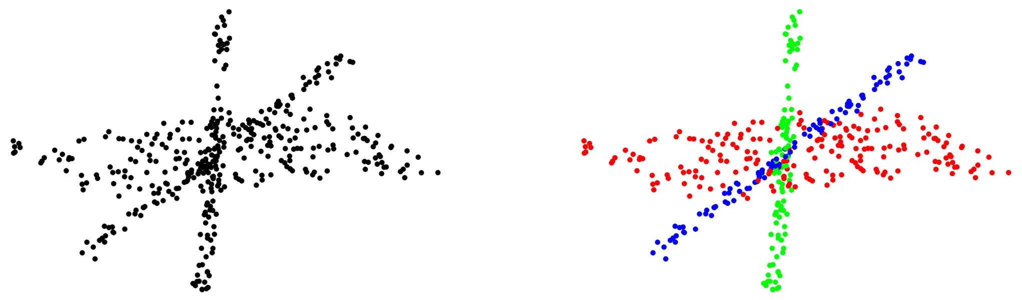
Modeling data by multiple low-dimensional planes is an important problem in many applications such as computer vision and pattern recognition. In the most general setting where only coordinates of the data are given, the problem asks to determine the optimal model parameters, estimate the model planes, and cluster the data accordingly. Though many algorithms have been proposed, most of them need to assume prior knowledge of the model parameters and thus address only the last two components of the problem. In this paper we propose an accurate and efficient algorithm based on multiscale SVD analysis and spectral methods to tackle the problem.

Problem Definition

We formulate two separate problems:

Problem 1. *(**Model Selection**) Given data $\mathbf{x}_1, \ldots, \mathbf{x}_N \in \mathbb{R}^D$ sampled around a collection of (unknown) K planes $\pi_1, \ldots, \pi_K$ of dimensions $d_1, \ldots, d_K$, determine the model parameters K, $(d_k)_{k=1}^K$ and $\{\pi_k\}_{k=1}^K$.*

Problem 2. *(**Subspace Clustering**) With the same data as in Problem 1 and prior knowledge on the model parameters $K, (d_k)_{k=1}^K$, cluster the data into K groups corresponding to the (unknown) model planes $\{\pi_k\}_{k=1}^K$.*



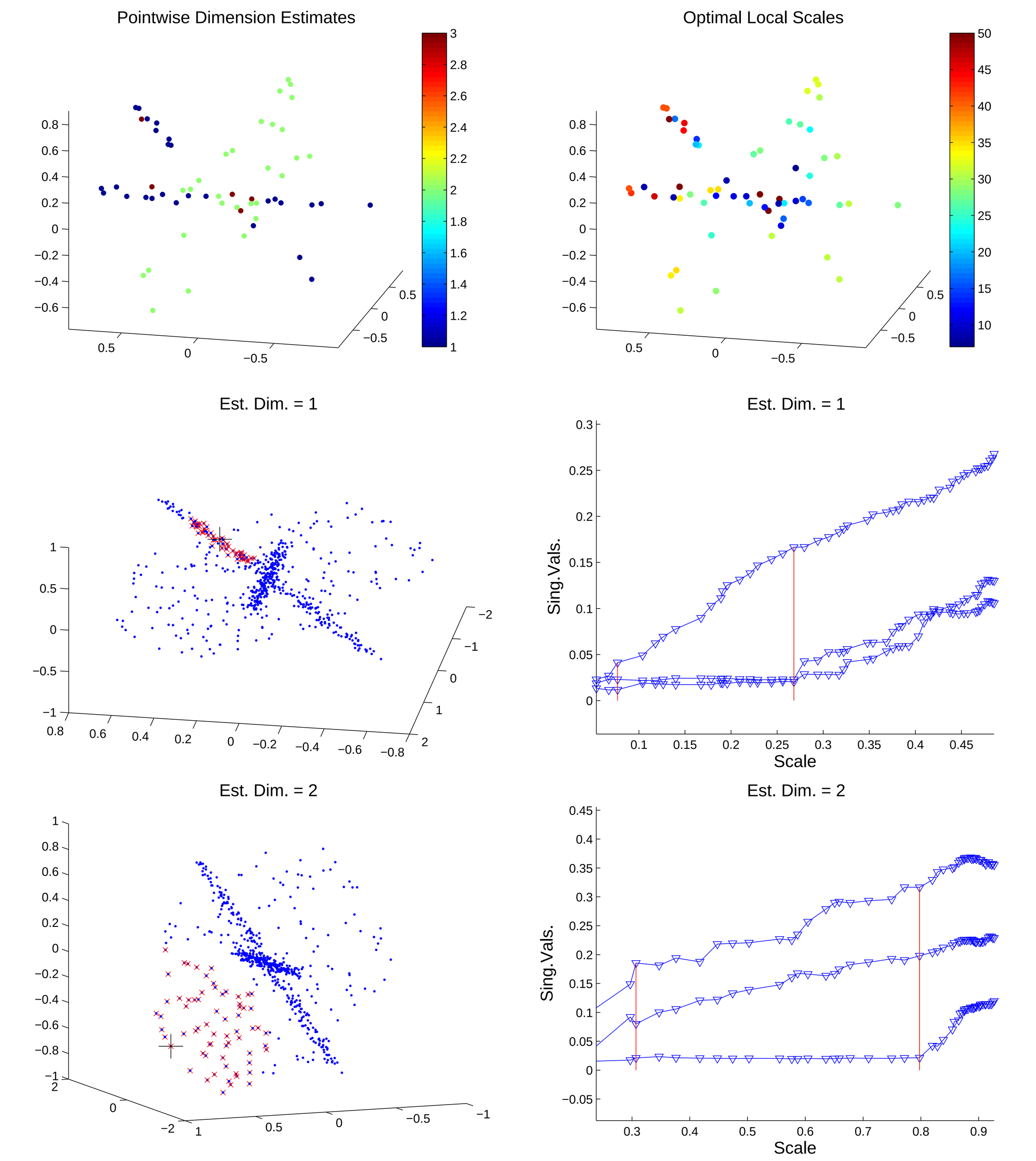
Remarks:

- Problem 1 has a different complexity than 2.
- Current literature focuses on Problem 2.

Methodology

Our approach (MAPA) has two stages:

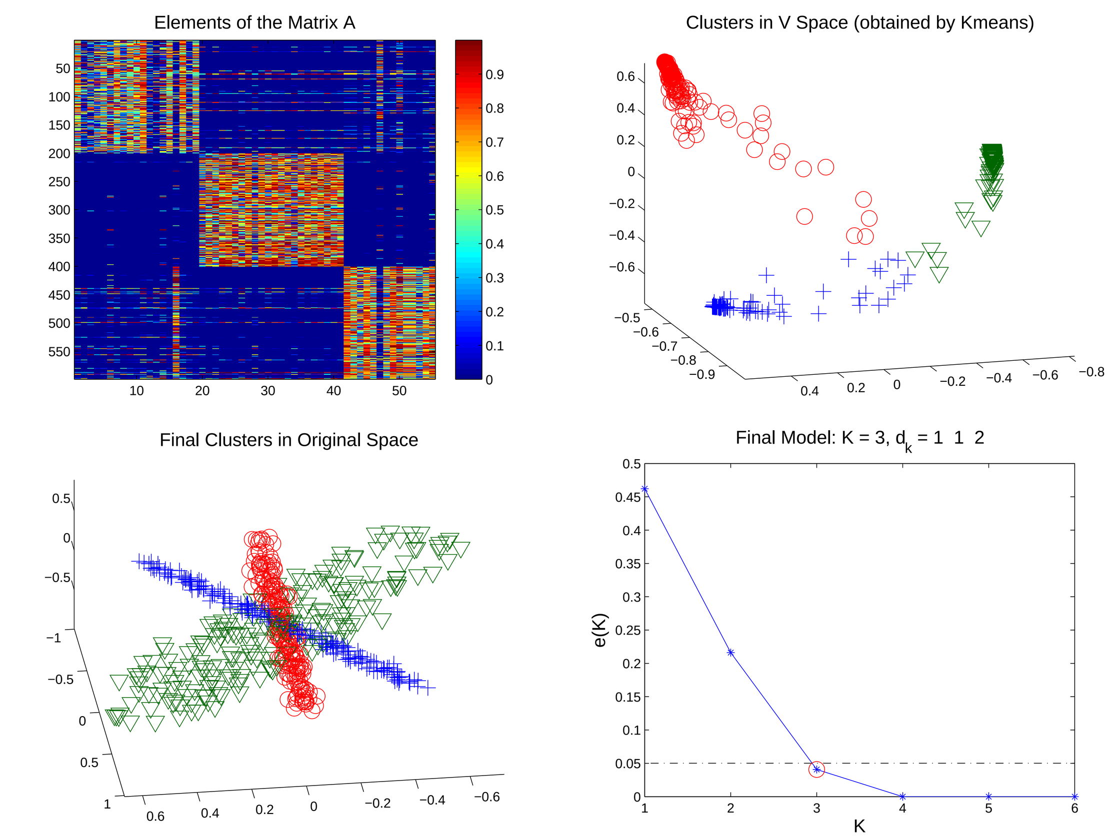
(I) Geometric Analysis. Multiscale SVD is applied to the data for estimating local dimensions $\hat{d}_j$, planes $\hat{\pi}_j$, errors $\hat{\epsilon}_j$, and optimal scales.



(II) Spectral Analysis. Points are embedded twice, first using the local planes:

$$\mathbf{A}_{ij} := e^{-\operatorname{dist}^2(\mathbf{x}_i, \hat{\pi}_j)/2\hat{\epsilon}_j^2}$$

and then by SVD, before applying Kmeans.



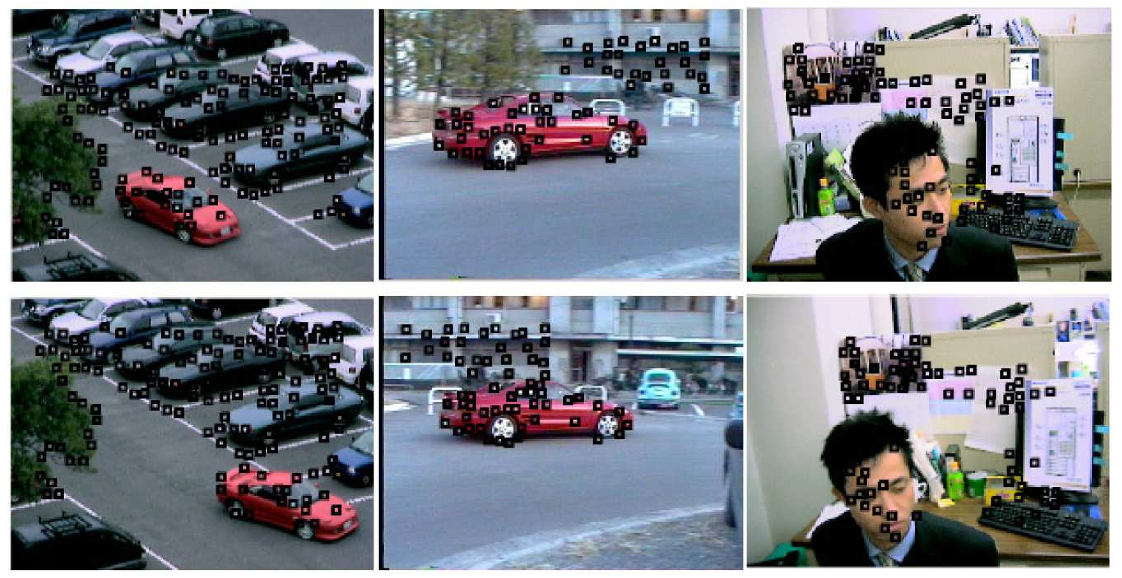
Theorem. *Given upper bounds $K_{\max}, d_{\max}$ and by accessing $O(d_{\max} K_{\max} \log K_{\max} \log d_{\max})$ samples, the MAPA algorithm returns w.h.p. the correct model parameters $(K; d_1, \ldots, d_K)$, and accurate approximations to $\{\pi_k\}_{k=1}^K$, in time $O(D K_{\max} d_{\max}(d_{\max} + K_{\max}) \log K_{\max} \log d_{\max})$.*

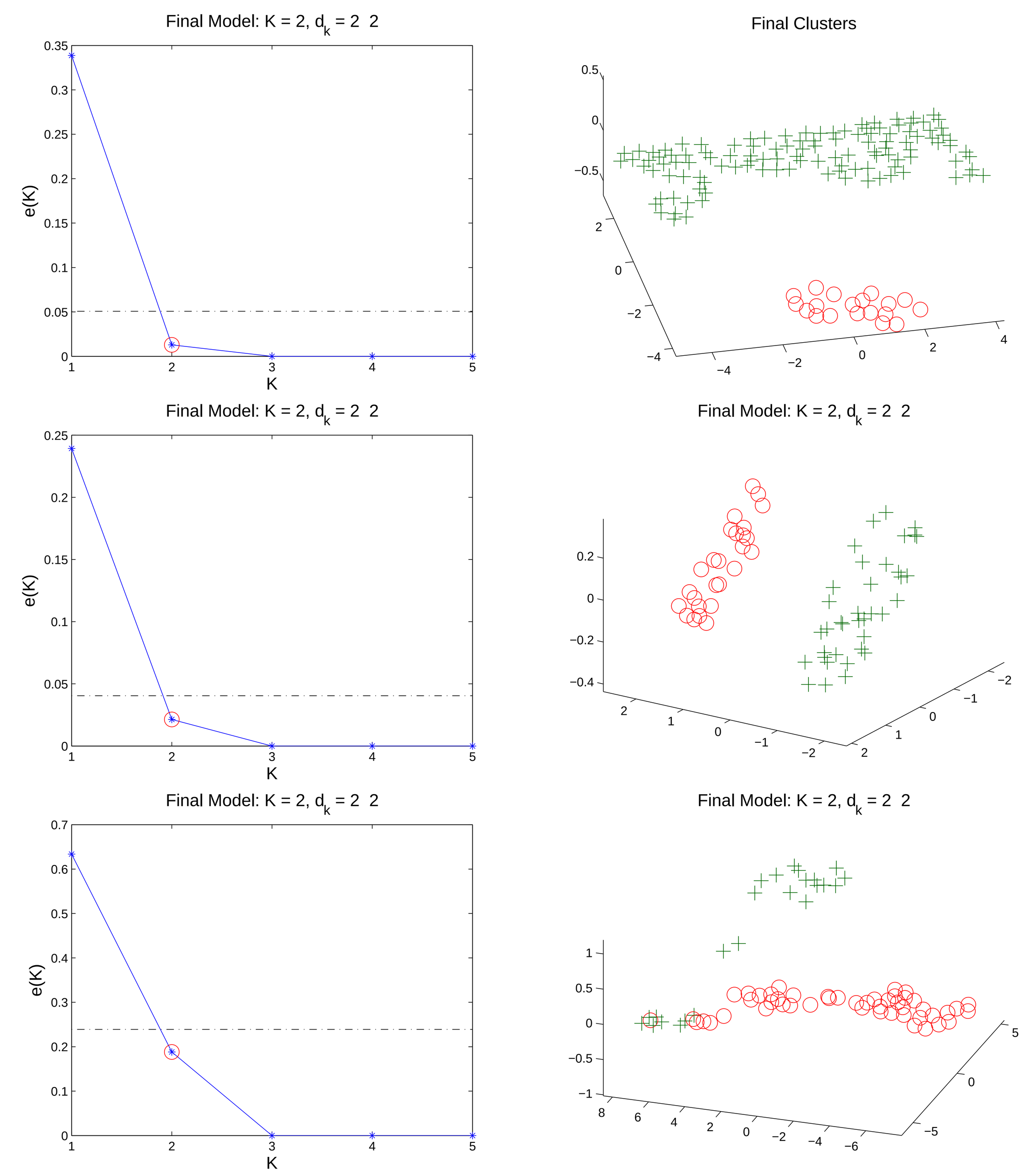
Applications

The algorithm could be applied to problems wherever one needs to model data using a union of subspaces. In this paper, we study the following two:

(I) Motion segmentation with affine camera models.

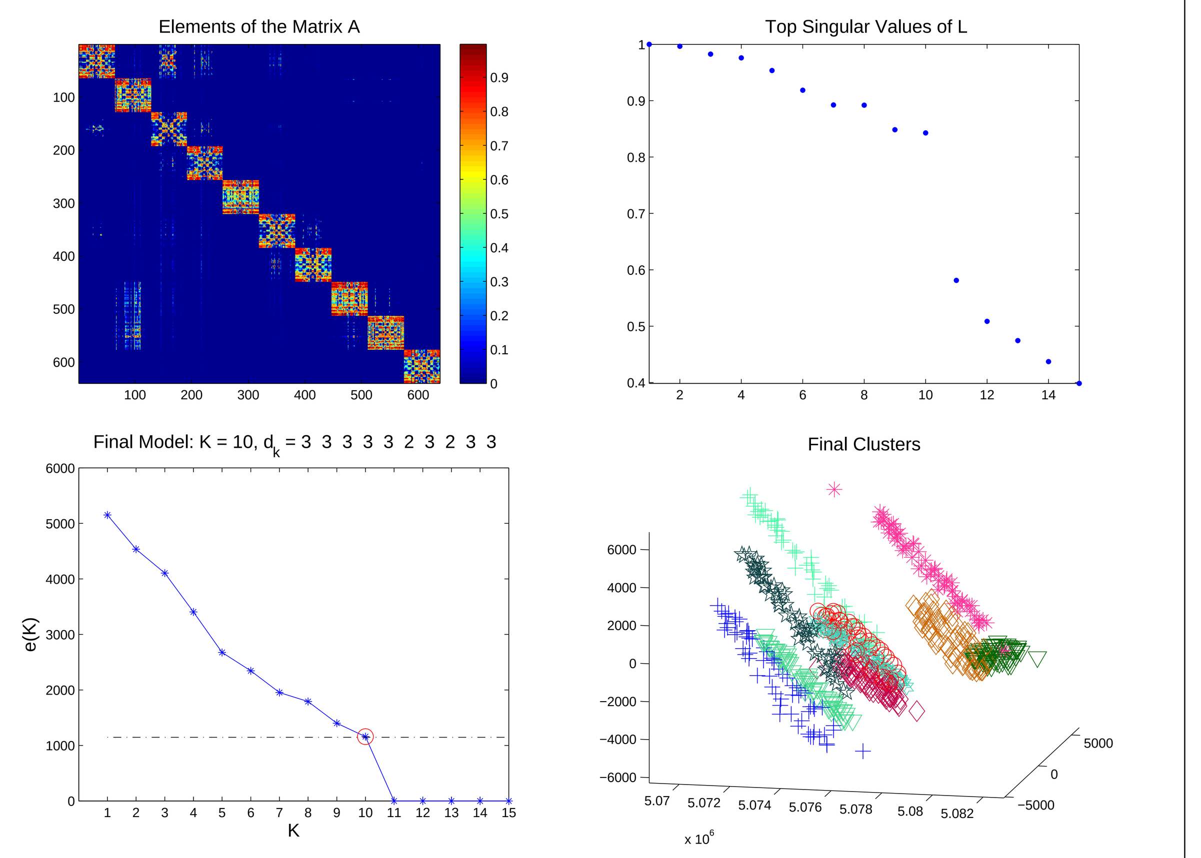
Three Kanatani motion sequences (only first and last frames shown in each column).





(II) Clustering of facial images in fixed pose under varying illumination angles.

Yale Face Database B. 10 subjects, frontal pose, 64 illumination angles.



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