Homework 4 - Due Oct. 10th
High-Dimensional Approximation, Probability, and Statistical Learning

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Homework Policies. As for the first homework set.

Assignment

Study: Introduction and Lecture 1 (up to what we covered in class) of P. Rigoleet's Lecture notes `http://www-math.mit.edu/~rigollet/PDFs/RigNotes15.pdf`.

Exercises.

Exercise 1 (30pts). Prove in detail Corollary 1.7. Interpret and re-write its statement in terms of projections of the random *vector* $(X_1, \ldots, X_n)$ in the direction of the vector a.

Exercise 2 (50pts). Give an estimate of the subGaussian variance proxy σ^2 of the following random vectors, paying particular attention at how it scales (or not) with the dimension D:

- $X \sim \mathrm{Unif}(\mathbb{B}_0^D(1))$, where $\mathbb{B}_0^D(1)$ is the unit ball in $\mathbb{R}^D$;
- $X \sim \mathrm{Unif}(\mathbb{S}_0^D(1))$, where $\mathbb{S}_0^D(1)$ is the unit sphere in $\mathbb{R}^D$;
- $X \sim \mathrm{Unif}([-\frac{1}{2}, \frac{1}{2}]^D)$;
- $X \sim \mathcal{N}(\mu, \Sigma)$, where $\mu \in \mathbb{R}^D$ and Σ is the covariance matrix (of this normal distribution).

Using your results for case 1 or 2 above (and exercise 1 and/or results discussed in class), obtain bounds, similar to the ones we discussed time ago in class, showing that most points uniformly distributed on the ball (or the unit sphere if you prefer) are close to "the" equator.

Exercise 3 (20 pts). Using Hoeffding's inequality, bound the probability of getting at least $0.75n$ tails when tossing n times a fair coin.