Homework 6 - Due Nov. 7th
High-Dimensional Approximation, Probability, and Statistical Learning

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Homework Policies. As for the first homework set.

Exercises.

Exercise 1 (60pts). Prove the following properties of the width $w(K)$ of a set K:

- $w(K)$ is finite if and only if K is bounded (i.e. it is contained in $B_2(R)$ for some $R > 0$);
- $w(\mathrm{conv}(K)) = w(K)$, where $\mathrm{conv}(K)$ is the convex hull of K, i.e. the smallest set containing K;
- $w(\alpha K) = |\alpha| w(K)$, where $\alpha K = \{\alpha x, \text{ for } x \in K\}$.
- if A is a rotation (more generally, a unitary, i.e. distance-preserving, linear transformation) and $t \in \mathbb{R}^n$, then $w(AK + t) = w(K)$, i.e. the width is independent of affine distance-preserving transformations.

Exercise 2 (40pts). Estimate $w(B_\infty^{\mathbb{R}^n})$, the width of the unit ℓ^∞ ball in $\mathbb{R}^n$. You can do this in various ways, but one way is by observing that $w(B_\infty^{\mathbb{R}^n}) = \mathbb{E}||\eta||_1$ (prove this, do not assume it!), with η standard Gaussian vector in $\mathbb{R}^n$.