High-Dimensional Approximation, Probability, and Statistical Learning

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The course covers fundamental mathematical ideas for certain approximation and statistical learning problems in high dimensions. We start with basic approximation theory in low-dimensions, in particular linear and nonlinear approximation by Fourier and wavelets in classical smoothness spaces, and discuss applications in imaging, inverse problems and PDE's. We then introduce notions of complexity of function spaces, which will be important in statistical learning. We then move to basic problems in statistical learning, such as regression and density estimation. The interplay between randomness and approximation theory is introduced, as well as fundamental tools such as concentration inequalities, basic random matrix theory, and various estimators are constructed in detail, in particular multi scale estimators. At all times we consider the geometric aspects and interpretations, and will discuss concentration of measure phenomena, embedding of metric spaces, optimal transportation distances, and their applications to problems in machine learning such as manifold learning and dictionary learning for signal processing.

Synopsis

- Basic approximation theory in low-dimensions: linear and nonlinear approximation by Fourier and wavelets in classical smoothness spaces, Sobolev spaces, and Besov spaces. Notions of complexity of function spaces. Introduction to applications in imaging, inverse problems and PDE's.

- Basic problems in statistical learning: regression, density estimation. The interplay between randomness and approximation theory.

- Curse of dimensionality: geometric aspects, and manifestations in approximation theory.

Geometric Aspects

- Singular value decomposition (SVD), low-rank matrices. First applications of SVD: dimension reduction, variances and covariances. Data and matrix compression; applications to computation.

- Concentration of measure phenomena. Basic concentration inequalities. Random matrices and their spectra, relationships with their continuous limits. Applications of random matrices to covariance matrix estimation, signal processing, compressed sensing.

- Dimension reduction. Random projections. Johnson-Lindenstrauss Lemma.

- Introduction to metric spaces. Mappings between metric spaces, Lipschitz maps, distortion; embeddability of metric spaces into Euclidean space (positive and negative results), approximation of metric spaces by trees. Applications to algorithms and to statistics.

- Clustering problems: K-means, spectral clusterings, connections with graph problems and random matrix problems. Diffusion processes on graphs and their applications.

Learning Functions

- Regression: problem statement, examples. Curse of dimensionality in regression: lower bounds. Linear regression: least squares. Regularization. Nonparametric and multiscale methods. Regression of Lipschitz and Hölder functions. Fourier and multiscale methods. Regression on manifolds. Adaptivity.

- Estimating probability measures and densities. Density Estimation. Spaces of probability measures. Optimal transportation distances. Estimation of singular measures. Multiscale methods.

Prerequisites

Analysis I & II; basic probability.

Recommended but not required: probability beyond the basics, or introduction to statistics; functional analysis. Basic programming skills in Matlab or C or R.