Homework 2 - Math 225
Due Thursday, Jan. 29

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Office hours: by appointment.

www.math.duke.edu/~ mauro/teaching.html

I prefer homework written in pen rather than pencil. The handwriting and organization of your work on the page should be clear. Include appropriate explanations for what you are doing in your calculations and why, and what conclusions you draw or observations you make.

The homework should include a printout of the Matlab/C/Fortran code you used and of the code output (including figures as needed/requested). Also send me a copy of the code via e-mail: if you have multiple files, compress them into a unique zip file. Name the file as FamilyName_FirstInitial_Homework_xx.zip, where xx is the homework number. This will apply to all the future homework as well. Please use the subject "Math 225 homework" in your e-mail.

1. This problem is based on Problem 43 on page 194 of Atkinson. Write a program from scratch to compute the DFT given by either Eq. 3.8.20 on page 181 of Atkinson:

$$d_k = \frac{1}{m} \sum_{j=0}^{m-1} W_m^{jk} f_j \,,$$

where $W_m = e^{-\frac{2\pi i}{m}}$, or the formulas we used in class from Quarteroni's book (similar to the one above, but with $N = m$ and W_N phase-shifted). In your program, you will need to keep track of both the real and complex sums.

Compute the DFT of order m of the following:

(a) $x_k = 1$, $k = 0, \ldots, m-1$
(b) $x_k = (-1)^k$, $k = 0, \ldots, m-1$
(c) $x_k = k$, $k = 0, \ldots, m-1$

Do part (a) both by explicit analysis and by means of your program for the DFT (this will provide a check, in part, of your DFT program). When you compute by means of your program, use $m = 10$. (You should think of x_k as $x_k = f_k = f(t_k)$, where $t_k = 2\pi k/m$, and where k is an dummy index, i.e., it could just as well be j.) Explain the answer that you obtain.

Do part (b) by means of your program, with $m = 10$. Produce a plot of d_k as a function of k, and explain why it looks the way it does. (This is a discrete plot, so the points should not be connected by lines.)

(c) Do part (c) of Problem 43 by means of your program but with $m = 4000$, 8000, 16000, and 32000 (or larger multiples). Report the time taken to perform the calculations in each case, and confirm the m^2 cost estimate.

If you are using Matlab, compare with the cost of using the built-in Matlab DFT.

2. Problems 5 and 6 in Atkinson, page 240: let $f(x) \in \mathcal{C}^3([-\alpha, \alpha])$, for some $\alpha > 0$, and consider approximating it by the rational function

$$R(x) = \frac{a + bx}{1 + cx} \,.$$

To generalize the idea of the Taylor series, choose the constants a, b, c so that

$$R^{(j)}(0) = f^{(j)}(0) \quad j = 0, 1, 2 \,.$$

Is it always possible to find such an approximation $R(x)$? The function $R(x)$ is an example of a *Pade approximation* to $f(x)$.

Apply the results above to the case $f(x) = e^x$ and give the resulting approximation $R(x)$. Analyze its error on $[-1, 1]$ and compare it with the error for the quadratic Taylor polynomial.