Homework 3 - Math 225
Due Thursday, Feb. 5th

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Office hours: by appointment.

www.math.duke.edu/~ mauro/teaching.html

I prefer homework written in pen rather than pencil. The handwriting and organization of your work on the page should be clear. Include appropriate explanations for what you are doing in your calculations and why, and what conclusions you draw or observations you make.

The homework should include a printout of the Matlab/C/Fortran code you used and of the code output (including figures as needed/requested). Also send me a copy of the code via e-mail: if you have multiple files, compress them into a unique zip file. Name the file as FamilyName_FirstInitial_Homework_xx.zip, where xx is the homework number. This will apply to all the future homework as well. Please use the subject "Math 225 homework" in your e-mail.

1. Prove that the Cavalieri-Simpson rule has degree 3.

2. Let $f(x) = \mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-s^2}\, ds$. Write programs that you need from scratch; use double-precision arithmetic.

   You can check your results and estimate error by means of comparisons with published tables or by making use of the alternating series that one gets by expanding the integrand in its Maclaurin series and integrating term by term. In an alternating series, one knows from elementary calculus that the magnitude of the error is less than the magnitude of the first term of the series that is neglected. The alternating series is

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)n!}.$$

   (a) Use the composite trapezoidal rule and its error formula to estimate $f(2)$ with absolute error less than 10^{-8}.

   (b) Use the composite Simpson's rule and its error formula to estimate $f(2)$ with absolute error less than 10^{-8}.

   (c) Based on the number of subintervals required for parts (a) and (b), and on other pertinent considerations, how do the computational costs of the two methods compare?

   (d) Let n be the number of subintervals used in an application of either Simpson's rule or the Trapezoidal rule to find $f(2)$. For both methods, obtain plots of the base-ten logarithm of an estimate of the magnitude of the error, as a function of the base-ten logarithm of n. Relate the resulting plots to the expected orders of convergence and to the asymptotic error formulas. For sufficiently large n the effects of floating point arithmetic may become apparent.

   (e) Discuss whether your estimates of $f(2)$ in parts (a) and (b) are likely to have been significantly influenced by floating point errors.

   (f) How would you use Richardson extrapolation and the trapezoidal rule to estimate $f(2)$ with absolute error less than 10^{-8}?