Homework 4 - Math 225
Due Tuesday, Feb. 17th

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Office hours: by appointment.

www.math.duke.edu/~mauro/teaching.html

I prefer homework written in pen rather than pencil. The handwriting and organization of your work on the page should be clear. Include appropriate explanations for what you are doing in your calculations and why, and what conclusions you draw or observations you make.

The homework should include a printout of the Matlab/C/Fortran code you used and of the code output (including figures as needed/requested). Also send me a copy of the code via e-mail: if you have multiple files, compress them into a unique zip file. Name the file as FamilyName_FirstInitial_Homework_xx.zip, where xx is the homework number. This will apply to all the future homework as well. Please use the subject "Math 225 homework" in your e-mail.

1. Numerical differentiation may be used to gain information about data of unknown accuracy or quality, or about data from an unknown source. In the file

   `www.math.duke.edu/~mauro/Teaching/Math225/Homework_4_Data.zip`

   you will find five files of data that represent functions f_i ($i = 1, 2, 3, 4, 5$) defined on $[0, 1]$, sampled at regular intervals of width h. This files are in text format and can immediately be loaded in Matlab, C or Fortran.

   (a) Write a program that numerically approximates derivatives $f_i^{(n)}$ of these functions with error $O(h^2)$.

   (b) Identify the functions f_i by examining the results of successive applications of the numerical derivative. The identifications should include explicit formulas with explicit coefficients, where possible.

   (c) Also, write a program that uses the trapezoidal rule to approximate $F_i^{+1}(x) = 1 + \int_0^x f_i(s)\ ds$ on $[0, 1]$, $F_i^{+2} = 1 + \int_0^x F_i^{+1}(s)\ ds$, etc.

   (d) Can useful information be obtained by successive integrations? Why, or why not?

   (Hint: functions are taken from exponentials, trigonometric functions, polynomials, piecewise polynomials; some may contain noise)

2. Generate code that produces N points in the unit ball in $\mathbb{R}^d$ uniformly at random, by using the accept/reject procedure described in class: draw a d points $x_1, \ldots, x_d$ uniformly in $[-1, 1]$ (we will learn how to do this later! For now use library functions, such as `rand` in Matlab), let $x = (x_1, \ldots, x_d) \in \mathbb{R}^d$ (in fact, in the cube of side 2 centered at 0), and accept x if it is indeed in the unit ball. Your function takes N, d as parameters.

   (a) How can you use this function to compute the value of the volume of the unit ball? How large does N need to be in order for this technique to have good accuracy? In which sense has one to reasonably interpret "accuracy" in this context? What is the variance of the estimated volume when repeating the draw of N points? Is it what you would expect?

   (b) Use the algorithm you described in (a) to compute the volume of the unit ball for $d = 2, 4, 6, 8, 10, 20$, with $N = 1000$. Compare with the known answer for the true volume of the unit ball in $\mathbb{R}^d$. Plot and discuss the running time of the algorithm as a function of d, and explain what dependency on N you expect. [$d = 20$ may take a long time, if it does you can give up].

   (c) Plot the distribution of distances from the origin of the N points you have drawn in (b), and discuss.