Homework 5 - Math 225
Due Tuesday, Feb. 24th

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Office hours: by appointment.

www.math.duke.edu/~mauro/teaching.html

I prefer homework written in pen rather than pencil. The handwriting and organization of your work on the page should be clear. Include appropriate explanations for what you are doing in your calculations and why, and what conclusions you draw or observations you make.

The homework should include a printout of the Matlab/C/Fortran code you used and of the code output (including figures as needed/requested). Also send me a copy of the code via e-mail: if you have multiple files, compress them into a unique zip file. Name the file as FamilyName_FirstInitial_Homework_xx.zip, where xx is the homework number. This will apply to all the future homework as well. Please use the subject "Math 225 homework" in your e-mail.

1. Solve the following problems using Euler's (forward) method with stepsizes $h = 0.2, 0.1, 0.05$. Compute the error and relative error using the true answer. For selected values of x, observe the ratio by which the error decreases when h is halved.

   (a) $y'(t) = [\cos(y(t))]^2 \quad , \ t \in [0, 10], \quad y(0) = 0$, [true sol: $y(t) = \arctan(t)$]

   (b) $y'(t) = \frac{1}{1+t^2} - 2[y(t)]^2 \quad , \ t \in \mathbb{R}, \quad y(0) = 0$, [true sol: $y(t) = \frac{t}{1+t^2}$]

   (c) $y'(t) = te^{-t} - y(t) \quad , \ t \in [0, 10], \quad y(0) = 1$, [true sol: $y(t) = (1 + \frac{1}{2}t^2)e^{-t}$]

2. Consider the linear equation

$$y'(t) = \lambda y(t) + (1 - \lambda)\cos(t) - (1 + \lambda)\sin(t) \quad , y(0) = 1 \, .$$

   The true solution is $y(t) = \sin(t) + \cos(t)$. Solve the problem using Euler's method with several values of λ and h, for $x \in [0, 10]$ and comment on the results:

   (a) $\lambda = -1$; $h = 0.5, 0.25, 0.125$

   (b) $\lambda = 1$; $h = 0.5, 0.25, 0.125$

   (c) $\lambda = -5$; $h = 0.5, 0.25, 0.125, 0.0625$

   (d) $\lambda = 5$; $h = 0.5, 0.25, 0.125, 0.0625$