

Homework 6 - Math 225 Due Thursday, Mar. 19th

Instructor: Mauro Maggioni

Office: 293 Physics Bldg.

Office hours: by appointment.

www.math.duke.edu/~mauro/teaching.html

I prefer homework written in pen rather than pencil. The handwriting and organization of your work on the page should be clear. Include appropriate explanations for what you are doing in your calculations and why, and what conclusions you draw or observations you make.

The homework should include a printout of the Matlab/C/Fortran code you used and of the code output (including figures as needed/requested). Also send me a copy of the code via e-mail: if you have multiple files, compress them into a unique zip file. Name the file as `FamilyName_FirstInitial_Homework_xx.zip`, where `xx` is the homework number. This will apply to all the future homework as well. Please use the subject "Math 225 homework" in your e-mail.

1. Consider the family of linear multistep methods

$$u_{n+1} = \alpha u_n + \frac{h}{2}(2(1 - \alpha)f_{n+1} + 3\alpha f_n - \alpha f_{n-1})$$

where α is a real parameter.

- Analyze consistency and order of the methods as functions of α , determining the value α^* for which the resulting method has maximal order
- Study the zero-stability of the method with $\alpha = \alpha^*$, write its characteristic polynomial $\Pi(r; h\lambda)$ and draw its region of absolute stability by using MATLAB.

2. Solve the differential equation

$$Y'(x) = -50Y(x) + 51 \cos(x) + 49 \sin(x) \quad , \quad Y(0) = 1$$

for $x \in [0, 10]$. The true solution is $Y(x) = \sin(x) + \cos(x)$. Use the AB method of order 2, as well as the MATLAB function `ode45`, for different values of relative and absolute errors. Familiarize yourself with the `ode45` function in Matlab, and its siblings. Use stepsize $h = 0.1, 0.02, 0.01$. In each case, plot the approximate solution, the true solution, and the error (in \log_{10} scale).