Multiscale Reconstruction of Hyper Spectral Data

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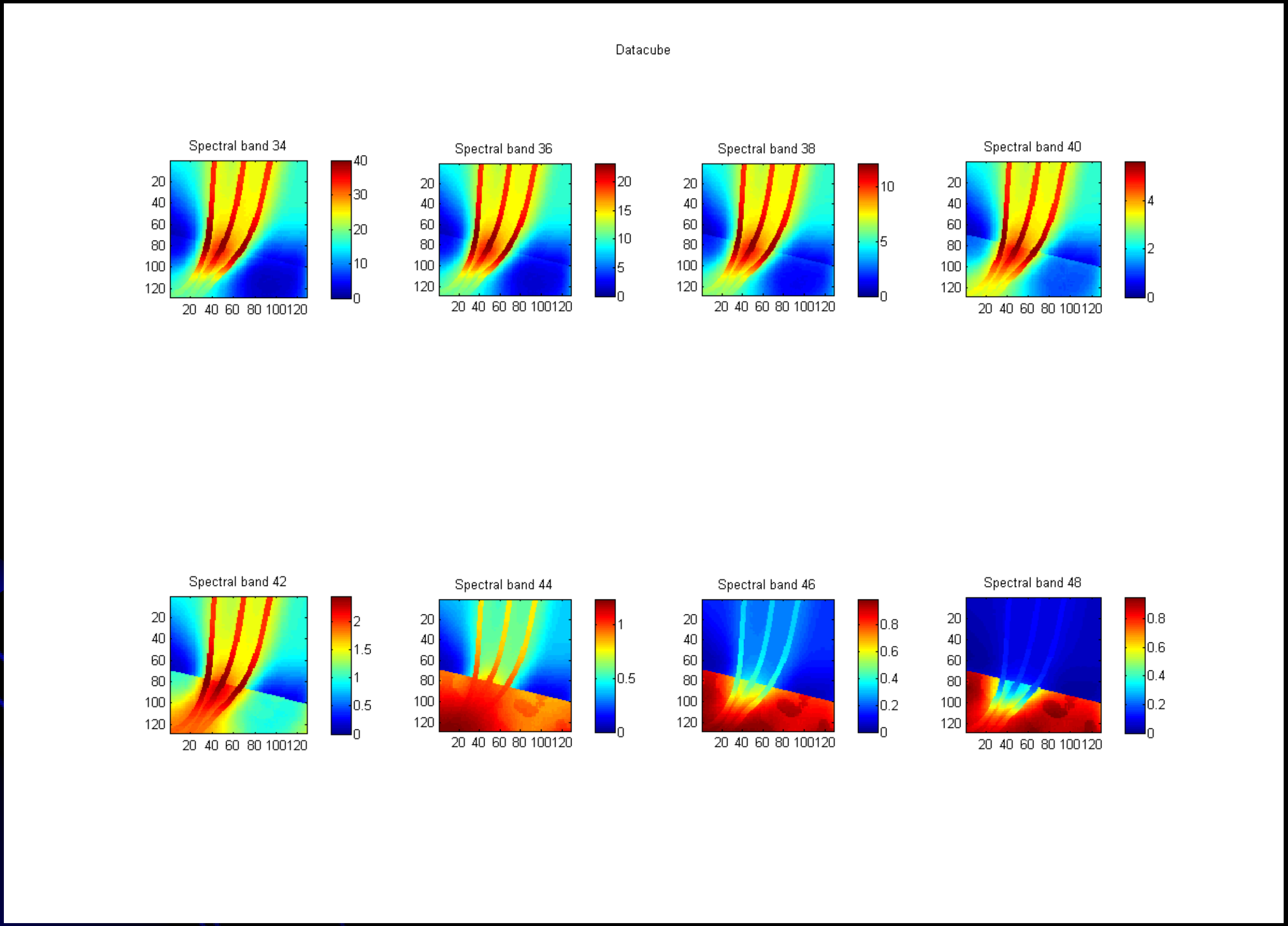
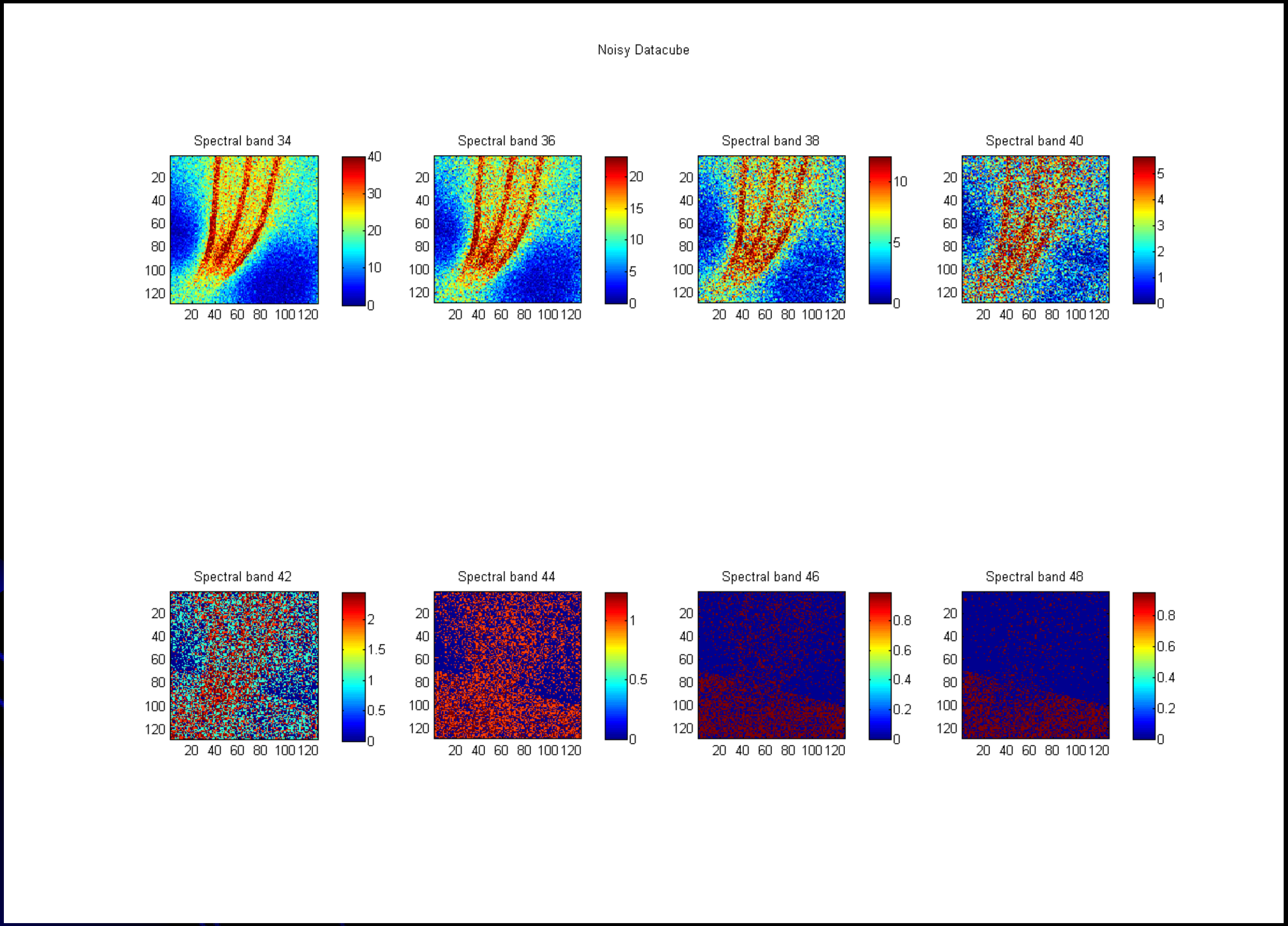
Why Hyper Spectral Imaging?



Problem 1

- Hyper Spectral Imaging does offer a lot of advantages over conventional imaging methods.
- However, it is not noise free.
- Often the data cube measured at the detector is noisy.
- Effect of noise on each slice of the data cube varies with photon counts.
- The problem of recovering the data cube from noisy observations is called Denoising.

Noisy Data cube

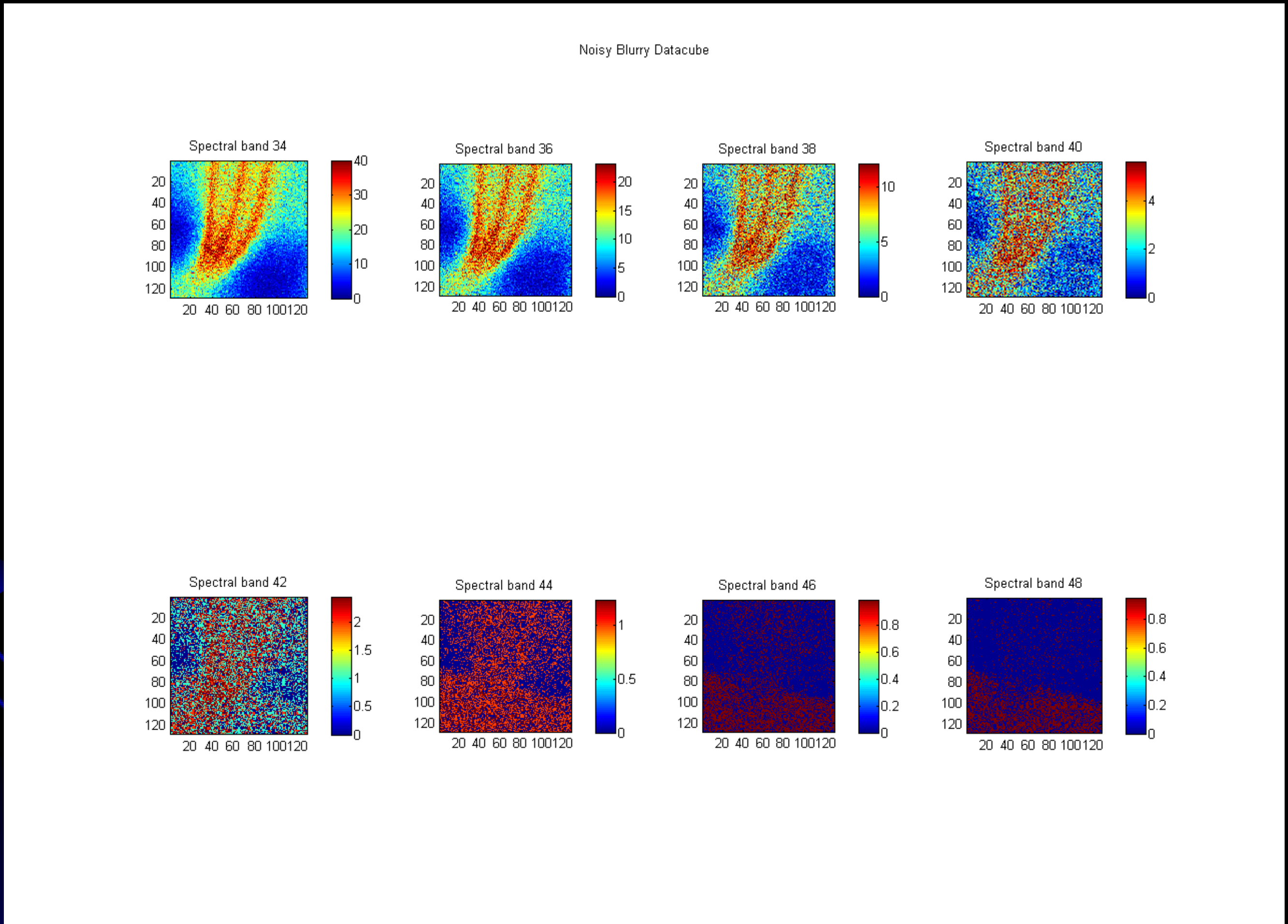


Problem 2

- Deblurring: Problem of recovering the data cube from the blur introduced by the optics and the noise introduced by the detector.

- Much more challenging than the denoising problem discussed before.

Noisy Blurry Data cube



Mathematical formulation of deblurring and denoising

- Denoising:
  - Let θ be the data cube.

$$y = poiss(\theta)$$

  - Given y, the problem is to recover θ.

- Deblurring:

$$y = poiss(H\theta)$$

  - H represents any blur that the optics might introduce.
  - Given y and H, the problem is to recover θ.

Multiscale approach to denoising

- The proposed multiscale denoising method comprises of two stages.

- Stage 1: Perform unnormalized Haar smoothing on the spatial dimensions.
  - Why unnormalized? Our observations are Poisson distributed with mean the true intensity. Sum of two Poisson variates is also Poisson distributed, however, the sum of two normalized Poisson variates is not Poisson. To preserve the Poisson distribution across scales, we resort to unnormalized Haar smoothing.

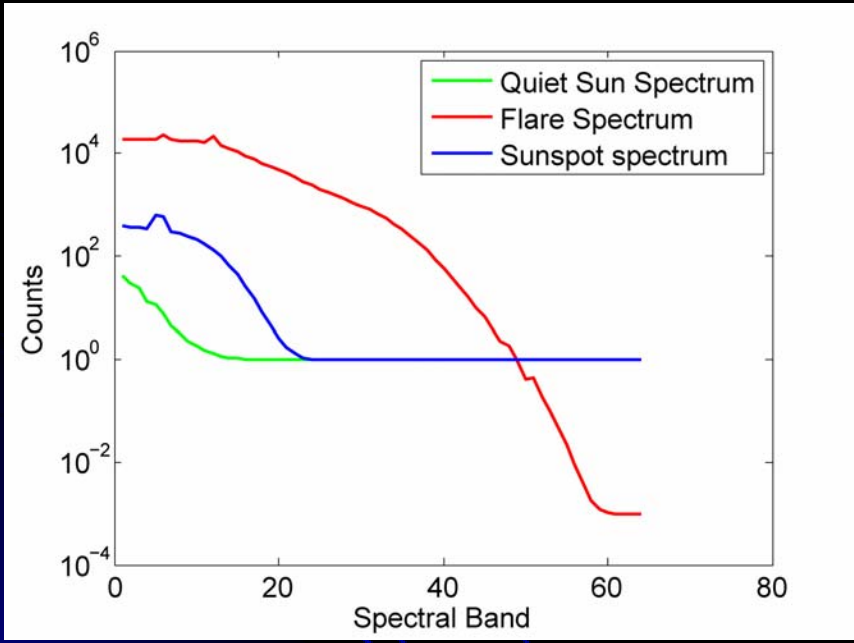
- Stage 2: Perform spectral smoothing. The kind of spectral smoothing to use depends on the structure of the spectrum.

  - We can use the same Haar smoothing for spectral dimension if the spectrum does not exhibit a very high dynamic rage.

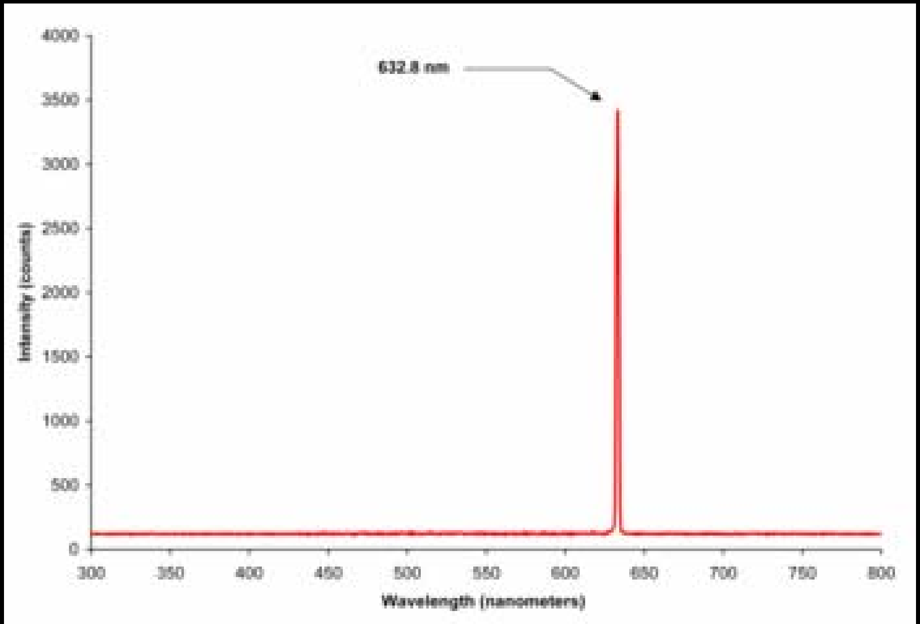
  - For a spectrum that has a wide dynamic range and an exponential decay, a multiscale generalized linear model fit might be more appropriate.

Spectra - Examples

Spectra exhibiting a very wide dynamic range – as seen in solar flares[1]



HeNe spectrum with a sharp peak at 632.8 nm

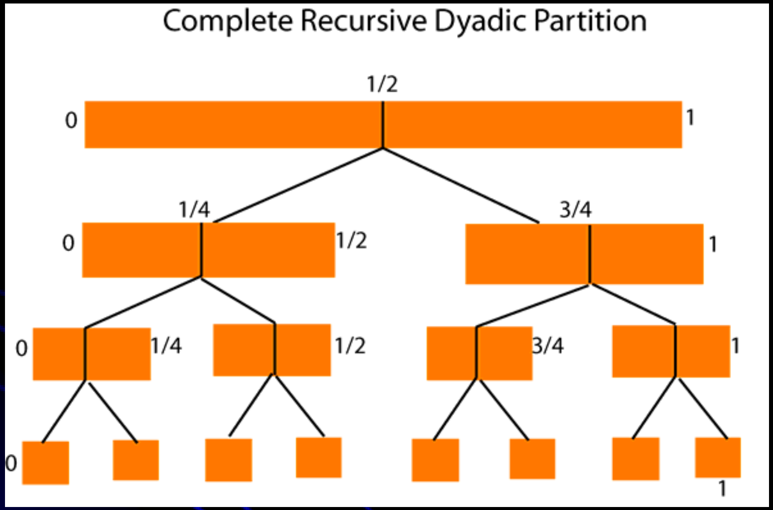


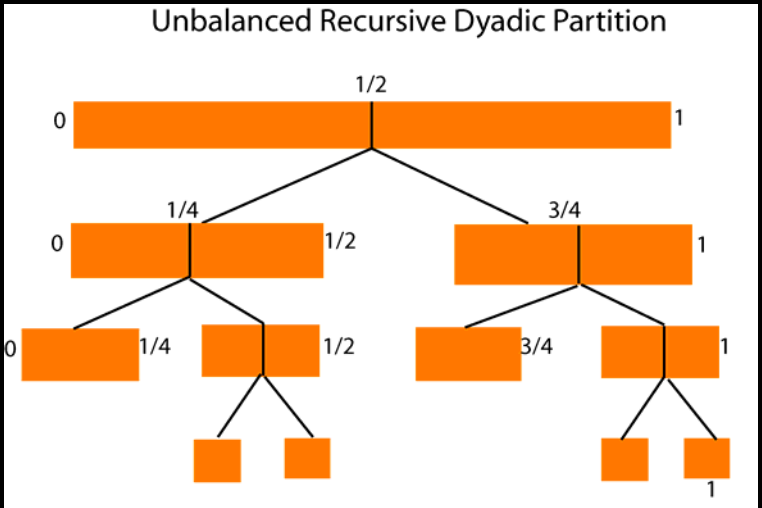
[1]R. P. Lin, B. R. Dennis, and A. O. Benz (Eds.), The Reuven Ramaty High-Energy Solar Spectrscopic Imager (RHESSI) – Mission Description and Early Results, Kluwer Academic Publishers, Dordrecht, 2003.

Hereditary Haar Poisson Intensity Estimation (1D case)

- Assume that we observe a signal in [0,1].
- Estimate the ideal partition of the observation using a maximum penalized likelihood estimation procedure.
- The space of possible partitions is a nested hierarchy defined through a recursive dyadic partition of [0 1].
- We prune the RDP based on the maximum penalized likelihood estimation and obtain an unbalanced binary tree representation as described by the following figures.

Recursive Dyadic Partition (RDP)





Maximum Penalized Likelihood Estimation (MPLE)

- Let the signal to be estimated be $X = \{x_i\}_{i=1}^{N}$
- Observed data: $Y = Poiss(X)$ $Y = \{y_i\}_{i=1}^{N}$
- Likelihood of the observed data given the true data

$$p(Y/X) = \prod_{i=1}^{N} \frac{e^{-x_i} x_i^{y_i}}{y_i!}$$

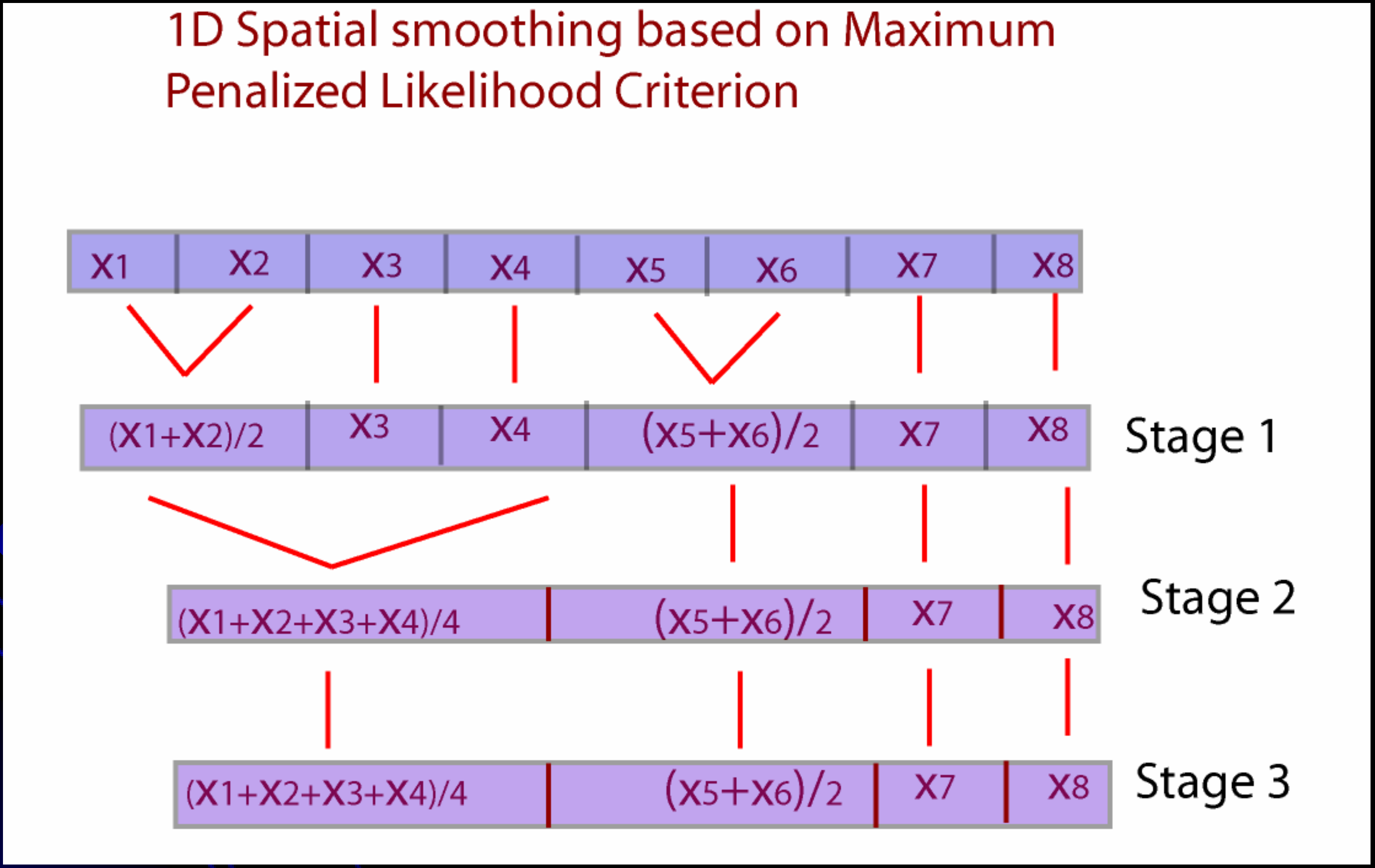
- We find the partition according to the equation

$$\hat{P} = \arg\min_{P}[-\log p(Y \mid X(P)) + pen(P)]$$

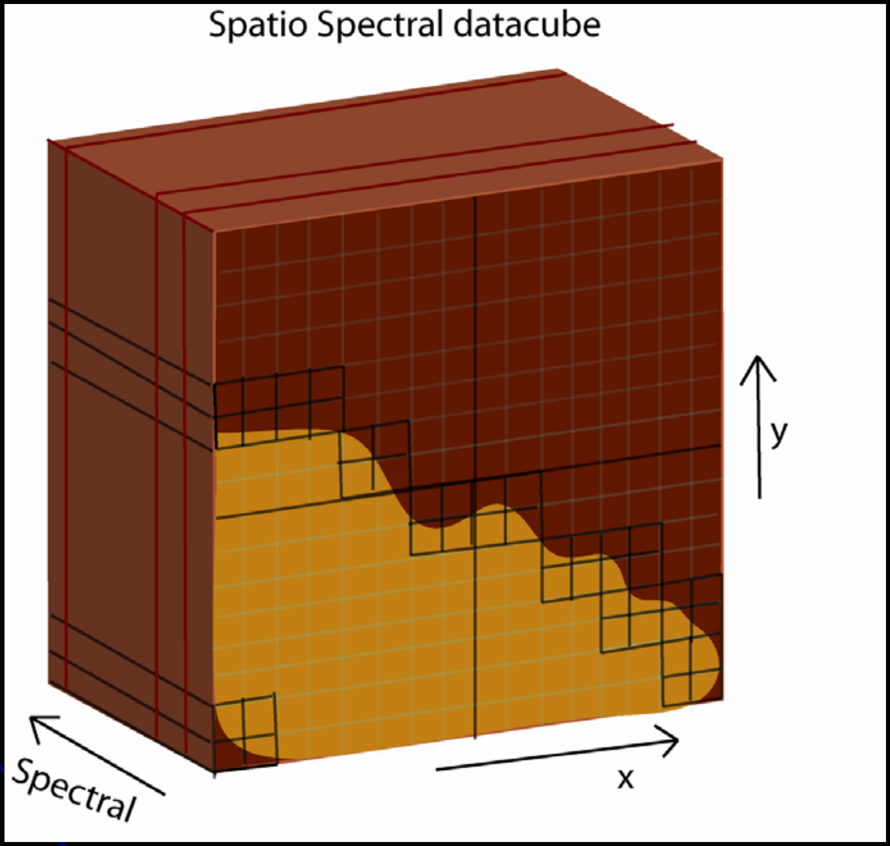
$$\hat{X} = X(\hat{P})$$

- The penalty term is proportional to the complexity of the partition. Higher penalty favors more smoothing and vice versa. As of now, choosing the right penalty is just a trial and error method.

1D multiscale denoising



Denoising 2D



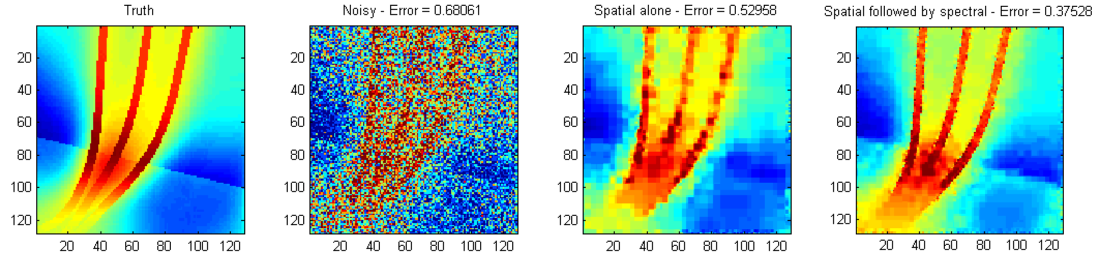
Denoising (contd…)

- Stage 1 described above does not consider the smoothness in the spectral dimension. Exploiting the spectral structure can improve the reconstruction accuracy by a great extent.

- Same penalty was used for both spatial and spectral smoothing and is given by

$$\frac{1}{5}\log\left(\sum_{i=1}^{N} y_i\right)$$

Denoising – Simulation results



Deblurring

- Let observed data cube be $y = Poiss(H\theta)$
- Deblurring step 1: Richardson Lucy algorithm

$$\tilde{\theta} = \theta^{t-1} .* H^T \left(y ./ H\theta^{t-1} \right)$$

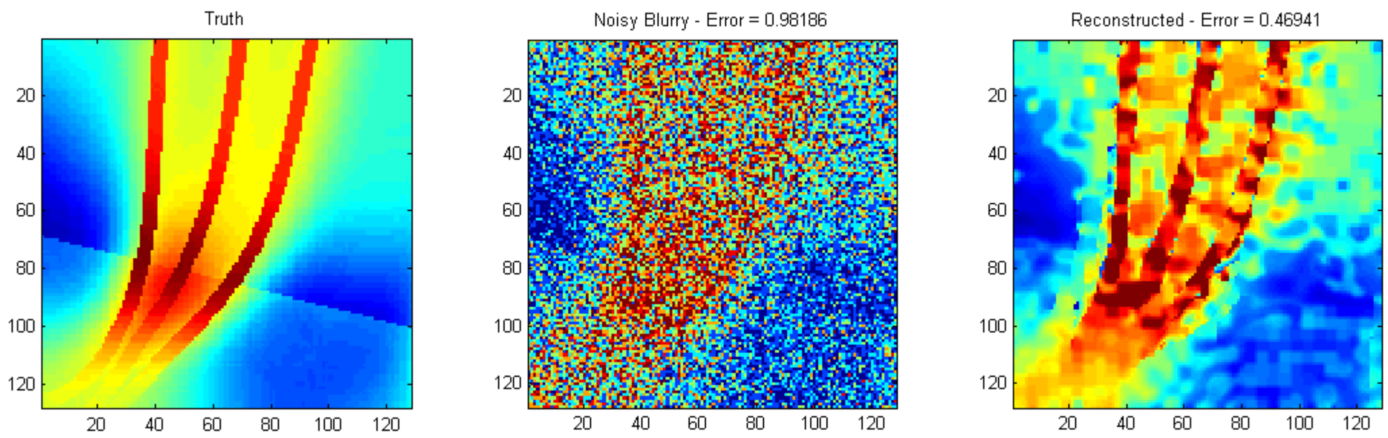
- Deblurring step 2: Denoise the estimate obtained in step 1 using the two stage denoising procedure described in the previous slides.

$$\theta^t = Denoise(\tilde{\theta})$$

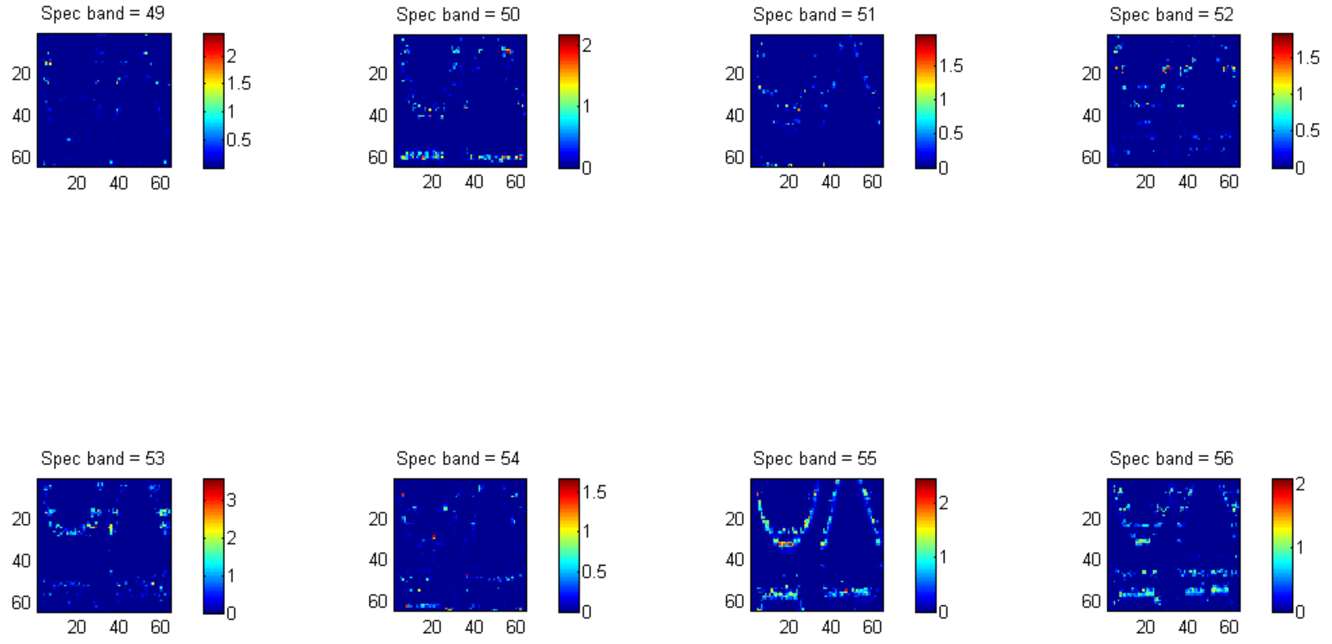
- Iterate steps 1 & 2 till the mean squared error between the current and the previous estimate becomes lesser than a set tolerance limit.

$$E[\theta^{t-1} - \theta^t]^2 \Big/ E[\theta^{t-1^2}] \leq \varepsilon$$

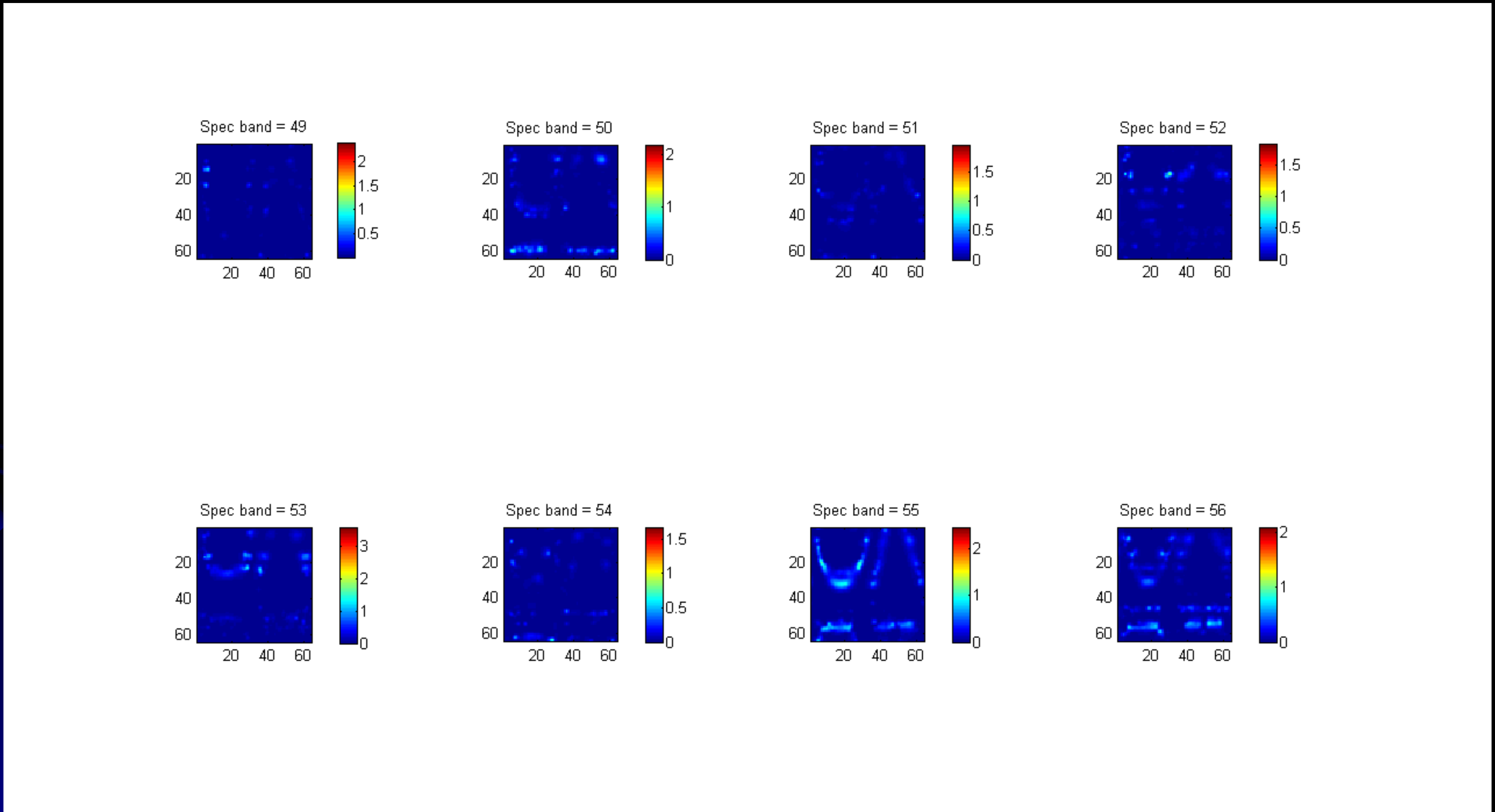
Deblurring – Simulation results



Application of the proposed approach to real data



Real data denoised using the proposed method



Spatial smoothing code from Dr. Rebecca Willett

Conclusion

- With the real data, the point spread function was not known. Hence, the deblurring algorithm could not be applied.
- Tuning in penalty for real data is much more complicated than the simulated data. Since we don't have ground truth available, there is no way to characterize the error and hence the performance of the proposed method.
- The test image was illuminated by a HeNe laser. So, the spectrum was nothing but a sharp peak at 632.8 nm with zero elsewhere. In such cases, the spectral smoothing might not offer any advantage over the spatial smoothing.
- Future work would involve obtaining the psf of the system and applying the proposed deblurring method to the real data.

References

- R. Willett, “Multiscale intensity estimation for marked Poisson processes,” in Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing — ICASSP, 2007.
- R. Willett, M.E.Gehm, D.J.Brady, “Multiscale reconstruction for computational spectral imaging”.