

Homework 2 - Math 431

Due Jan 22nd

Instructor	Mauro Maggioni
Office	319 Gross Hall
Office hours	1:30-2:30pm (usual time).
Web page	www.math.duke.edu/~mauro/teaching.html

Reading: from Reed's textbook: finish chapter 1.

Problems:

§1.1: #8, 10, 11 (in these problems assume only that x and y are elements of an ordered field \mathbb{F} ; in #11 assume in addition that \mathbb{F} is Archimedean. In the hint for #11, use the well-ordering principle to show that m exists)

§1.4: #9, 11 (You don't need a . to do c .; try using b . and #11 above)

Additional Problems:

1. Prove that the field \mathbb{C} of complex numbers cannot be given the structure of an ordered field. (*Suggestion:* Argue by contradiction: suppose a subset $P \subseteq \mathbb{C}$ exists with the required properties; then $i \in P \cup (-P)$, where i is the complex number such that $i^2 = -1$. Deduce the contradiction from this.)

2.. Let F be a field. Prove that if there is an integer $n \in \mathbb{N}$ such that $1 + 1 + \cdots + 1$ (n terms) $= 0$, then there is no subset $P \subseteq F$ satisfying the axioms of an ordered field. (It can be deduced from this that if (F, P) is an ordered field, then $\mathbb{Q} \subseteq F$.) Use this to prove that no finite field can be given the structure of an ordered field.

3. Prove that the Archimedean property does not hold in the ordered field $\mathbb{R}(x)$, by considering its two elements $\frac{1}{1}$ and $\frac{x^2}{1}$.

4. All horses are the same color: clearly, any set of one horse is the same color; assuming that in every set of n horses all are the same color, we conclude that every set of $n + 1$ horses, labeled from 1 to $n + 1$, has the same color, by considering the subsets of horses labeled from 1 to n and from 2 to $n + 1$, each of which must be the same color. Where's the flaw in this argument? (One possibility is that Mathematical Induction, hence WO, is flawed, or can't be applied here for some reason.)