

## Homework 4 - Math 431

### Due Feb 12th

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Office            319 Gross Hall  
Office hours    1:30-2:30pm  
Web page        [www.math.duke.edu/~mauro/teaching.html](http://www.math.duke.edu/~mauro/teaching.html)

**Reading:** from Reed's textbook: section 2.2-2.6.

**Problems:**

§2.1: #4b,6,9b

§2.2: #1b, 2a, 4, 5

§2.4: #1, 3, 5

§2.5: #1, 3

§2.6: #2,4,6

**Additional Problem:**

1. Show that  $\lim_{n \rightarrow +\infty} r^n$  does not exist if  $r \leq -1$ , by showing that for any  $L \in \mathbb{R}$ , the statement

$$\lim_{n \rightarrow \infty} r^n = L$$

is false. [*Suggested steps:* after carefully stating what you want to prove, take  $\epsilon = \frac{1}{4}$  in your statement. If  $|L - r^n| > \frac{1}{4}$  (what is  $n$ ?), you are good. If not, notice that  $|r^n - r^{n+1}| \geq 1$  (why?), then use the inequality of #10, §1.1 to conclude that  $|L - r^{n+1}| > 1/4$ ).

2. A real number  $d$  is said to be a *limit point* of a sequence  $\{a_n\}$  if for any  $\epsilon > 0$  and any  $N \in \mathbb{N}$ , there exists  $n \geq N$  such that  $|a_n - d| \leq \epsilon$ , or in logical form,  $(\forall \epsilon > 0)(\forall N \in \mathbb{N})(\exists n)(n \geq N \wedge |a_n - d| \leq \epsilon)$ . Write the logical and then the prose form of the statement: " $d$  is not a limit point of  $\{a_n\}$ ".