

## Homework 9 - Math 431

### Due Apr 9th

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| Instructor   | Mauro Maggioni   |
| Office       | 319 Gross Hall   |
| Office hours | 1:30-2:30pm  |
| Web page     | <a href="http://www.math.duke.edu/~mauro/teaching.html">www.math.duke.edu/~mauro/teaching.html</a> |

#### Problems:

§4.3: #11(a,e)

§4.6: #5(c,d,e) (give reasons)

§5.1: #1,4,11

§5.2: #9,15

#### Additional Problems:

1. In §4.3: #11(a) above you showed that  $\ln x \rightarrow \infty$  as  $x \rightarrow \infty$ . Use a similar argument to show that

$$\int_{\frac{1}{n}}^1 \frac{dt}{t} \rightarrow \infty$$

as  $n \rightarrow \infty$ . (Pick an appropriate partition  $P$  of  $[\frac{1}{n}, 1]$  and show that the corresponding lower sum tends to  $\infty$  as  $n \rightarrow \infty$ .) Conclude that  $\ln x \rightarrow -\infty$  as  $x \rightarrow 0$ .

2. Let  $\{f_n(x)\}$  be a sequence of functions which converges pointwise to a function  $f(x)$  on some subset  $E \subseteq \mathbb{R}$ . Suppose there exists a sequence  $\{x_n\} \subseteq E$  and a positive number  $c$  such that  $|f_n(x_n) - f(x_n)| > c$ , for all  $n$ . Prove that  $\{f_n(x)\}$  does not converge uniformly to  $f(x)$  on  $E$ .