Homework 2 - Math 431
Due Jan 25th/26th (section 1 and 2 respectively)

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Reading: from Reed's textbook: review section 1.1, study section 1.4, look ahead at section 1.2.

Problems:

§1.1: #3, 4, 7 [10 pts each]

§1.1: #8 [10 pts], 11 [20 pts] (in these problems assume only that x and y are elements of an ordered field $\mathbb{F}$; in #11 assume in addition that $\mathbb{F}$ is Archimedean. In the hint for #11, use the well-ordering principle to show that m exists)

§1.4: 11(*) (You don't need *a.* to do *c.*; try using *b.* and #11 above)

Additional Problems:

1. Prove that the field $\mathbb{C}$ of complex numbers cannot be given the structure of an ordered field. (*Suggestion:* Argue by contradiction: suppose a subset $P \subseteq \mathbb{C}$ exists with the required properties; then $i \in P \cup (-P)$, where i is the complex number such that $i^2 = -1$. Deduce the contradiction from this.) [10 pts]

2. Let F be a field. Using only the axioms (for a field), prove that $-ab = (-a)b$ for all $a, b \in F$. [10 pts]

3.. Let F be a field. Prove that if there is an integer $n \in \mathbb{N}$ such that $1 + 1 + \cdots + 1$ (n terms) $= 0$, then there is no subset $P \subseteq F$ satisfying the axioms of an odered field. (It can be deduced from this that if (F, P) is an ordered field, then $\mathbb{Q} \subseteq F$.) Use this to prove that no finite field can be given the structure of an ordered field. [10 pts]

4. Prove that the Archimedean property does not hold in the ordered field $\mathbb{R}(x)$, by considering its two elements $\frac{1}{1}$ and $\frac{x^2}{1}$. [10 pts]