Homework 5 - Math 431
Due Feb 15th

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Reading: from Reed's textbook: all of chapter 2, then section 3.1.
Problems:
§2.5: #1, 2, 3 [10 pts each]
§2.6: #1, 2, 4 [10 pts each], 6 [20 pts], 7 (*)

Additional Problem:
1. Show that $\lim_{n\to+\infty} r^n$ does not exist if $r \leq -1$, by showing that for any $L \in \mathbb{R}$, the statement

$$\lim_{n\to\infty} r^n = L$$

is false. [*Suggested steps*: after carefully stating what you want to prove, take $\epsilon = \frac{1}{4}$ in your statement. If $|L - r^n| > \frac{1}{4}$ (what is n?), you are good. If not, notice that $|r^n - r^{n+1}| \geq 1$ (why?), then use the inequality of #10, §1.1 to conclude that $|L - r^{n+1}| > 1/4$). [10 pts]

2. A real number d is said to be a *limit point* of a sequence $\{a_n\}$ if for any $\epsilon > 0$ and any $N \in \mathbb{N}$, there exists $n \geq N$ such that $|a_n - a| \leq \epsilon$, or in logical form, $(\forall \epsilon > 0)(\forall N \in \mathbb{N})(\exists n)(n \geq N \wedge |a_n - a| \leq \epsilon)$. Write the logical and then the prose form of the statement: "d is not a limit point of $\{a_n\}$". [10 pts]