Homework 6 - Math 431
Due Feb 22nd

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Studying: from Reed's textbook: section 3.1, 3.2
Problems:
§3.1: #3, 7, 8, 10. [10, 10, 20, 20 pts]
#5 (*).

Additional Problem:

1. Let $\{a_n\}$ and $\{b_n\}$ be Cauchy sequences. Let $\{a_n\} \sim \{b_n\}$ mean that $a_n - b_n \to 0$. Prove that $\sim$ is an equivalence relation: $\{a_n\} \sim \{a_n\}$; if $\{a_n\} \sim \{b_n\}$ then $\{b_n\} \sim \{a_n\}$; if $\{a_n\} \sim \{b_n\}$ and $\{b_n\} \sim \{c_n\}$, then $\{a_n\} \sim \{c_n\}$. [10 pts]

2. Prove that the sum and product of Cauchy sequences is Cauchy. [10 pts]

3. Let $[a_n]$ denote the equivalence class of the Cauchy sequence $\{a_n\}$. Given Cauchy sequences $\{a_n\}$ and $\{b_n\}$, define the sum and product of the equivalence classes containing them by

$$[a_n] + [b_n] := [a_n + b_n],$$

$$[a_n] \cdot [b_n] := [a_n b_n].$$

Prove that these rules are well-defined by showing that if $\{a_n\} \sim \{a'_n\}$ and $\{b_n\} \sim \{b'_n\}$, then $\{a_n + b_n\} \sim \{a'_n + b'_n\}$ and $\{a_n b_n\} \sim \{a'_n b'_n\}$. [10 pts]

4. If $\mathcal{C}$ denotes the set of equivalence classes of Cauchy sequences, then with the sum and product operations in 3. $\mathcal{C}$ is in fact a field. Don't try to prove this, but identify 0 and 1 in $\mathcal{C}$ and verify that $[a_n] + 0 = [a_n]$ and $[a_n] \cdot 1 = [a_n]$ for all Cauchy sequences $\{a_n\}$.(Keep in mind that your choice of 0 (or 1) in your answer will be an *equivalence class* of Cauchy sequences. This class may be identified by any Cauchy sequence in it.)
[10 pts]