Homework 4 - Due Mon. Feb. 27th
AS.110.446, EN.550.416

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Homework Policies

As in the first homework set.

Exercises

Exercise 1 (25 pts). If you only know that Q is a 3×3 orthogonal matrix, what can you say about x if the following equation holds?

$$Q \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ x \\ 0 \end{bmatrix}.$$

Exercise 2 (75 pts). First consider the following questions:

- Recall: $A^\dagger$ is the pseudo-inverse of $A \in \mathbb{R}^{m \times n}$, $m \geq n$, defined, for A full rank, by $A^\dagger = (A^T A)^{-1} A^T$. Write down $A^\dagger$ in terms of the SVD of A.
- Write the condition number $\kappa(A) = ||A|| \cdot ||A^\dagger||$ of the matrix A in terms of singular values of A.

Consider the code we discussed in class to perform least squares regression of a function with polynomials of degree up to d.

- Discuss the condition number $\kappa(A)$ of the matrix A in that code, as a function of n and d. Focus on the case when A is square ($n = d + 1$), and when n grows with d fixed.
- Let `W=randn(n,d+1)` be a random matrix, with entries independent and identically distributed according to a normal distribution. Consider the case $n \geq d + 1$. Study the condition number of W and compare it to that of A with similar size. Discuss.
- Let A and W be as above and square ($n = d + 1$). Let $b = \texttt{randn}(n, 1)$ be a random vector distributed as $\mathcal{N}(0, I_n)$ (why?), and solve $Ax_A = b$ and $Wx_W = b$. Now let $\eta = \texttt{randn}(0, \sigma^2)$, for $\sigma^2 = 0, 0.001, 0.01, 0.1$ and solve $A\tilde{x}_A = b + \eta$ and $W\tilde{x}_W = b + \eta$. Compute the perturbations $||x_A - \tilde{x}_A||/||x_A||$ and $||x_W - \tilde{x}_W||/||x_W||$. Discuss what you see, for different values of n and σ^2, and compare with $\kappa(A)$ and $\kappa(W)$. Say what this means in terms of the stability of the solution in terms of perturbations.