**Exercises for the last lectures**
**Math 561 - Fall 2015**

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This set of exercises cover some of the materials in the second part of the course. You do not have to turn them in; they are listed as suggestions for checking your understanding of the materials in the last few lectures. Of course, I will be happy to discuss any of these exercises or other materials during office hours.

From Trefethen's book: 31.4, 32.1, 33.2 (note that the exercise goes over to the next page, 256), 36.3, 36.4, 38.6
From Heath's book:

4.1 True or false:

- All eigenvalues of a matrix are not necessarily distinct
- A square matrix $A$ is singular if and only if 0 is one of its eigenvalues
- If two matrices have the same eigenvalues, then the two matrices are similar (i.e. if $A$ and $B$ are the two matrices, then there is a matrix $C$ such that $A = CBC^{-1}$)
- The condition number of the problem of solving a linear system with matrix $A$ is the same as the condition number of the problem of finding the eigenvalues of $A$
- For a square matrix, the eigenvalues and the singular values are the same

- Which of the following conditions necessarily imply that an $n \times n$ real matrix $A$ is diagonalizable?
  - $A$ has $n$ distinct eigenvalues
  - $A$ has only real eigenvalues
  - $A$ is nonsingular
  - $A$ is equal to its transpose
  - $A$ commutes with its transpose
- Give an example of a matrix which is not diagonalizable
- For which of the following classes of matrices of order $n$ can the eigenvalues be computed ina finite number of steps for arbitrary $n$?
  - diagonal
  - tridiagonal
  - triangular
  - Hessenberg
  - general real matrix with distinct eigenvalues
  - general real matrix with eigenvalues that are not necessarily distinct
- In using $QR$ iteration for computing the eigenvalues of a matrix, why is the matrix usually first reduced to some simpler form, such as Hessenberg or tridiagonal?
- Applied to a given matrix $A$, $QR$ iteration converges to either diagonal or triangular form: what determines which of the two forms is obtained?

- Why are shifts used in iterative methods for computing eigenvalues, such as in inverse iteration or $QR$ iteration?

- Show that eigenvectors corresponding to different eigenvalues of an Hermitian matrix are orthogonal.

- Implement power iteration to compute the dominant eigenvalues and corresponding normalized eigenvector of the matrix

$$A = \begin{bmatrix} 2 & 3 & 2 \\ 10 & 3 & 4 \\ 3 & 6 & 1 \end{bmatrix}.$$

  As starting vector take $[0; 0; 1]$ (a column vector). Then deflate out the eigenvalue you found and apply power iteration again to compute hte second largest eigenvalue of the same matrix. Compare your results to those you obtain with a standard library (e.g. Matlab's EIG).