Homework 3 - Due Wed. Sep. 16th
Math 561

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Homework policy as in the previous homework.

Assignment

Exercise 1 (35 pts). [Inspired by Ex. 5.3 in Trefethen's book] Consider the matrix

$$A = \begin{bmatrix} 3 & -10 \\ -2 & 2 \end{bmatrix}$$

- Compute, on paper, a real SVD of A in the form $A = U\Sigma V^T$. The SVD is not unique, so find the one with the minimal number of minus signs in U and V.
- List the singular values, left singular vectors, right singular vectors.
- Draw an accurate picture of the unit ball in $\mathbb{R}^2$ and its image under A, together with the singular vectors, with their coordinates marked.
- What are the $1-$, $2-$, $\infty-$ and Frobenius norms of A?
- What is the rank of A? How can it be read from the SVD?
- Find A^{-1} not directly, but via the SVD
- Find the eigenvalues λ_1, λ_2 of A
- Verify that $\det A = \lambda_1 \lambda_2$ and $|\det A| = \sigma_1 \sigma_2$.
- What is the area of the ellipsoid onto which A maps the unit ball of $\mathbb{R}^2$?
- What is the best rank-1 approximation to A?

Exercise 2 (15 pts). [Ex. 3.16 from Heath's book] Let A be a $m \times n$ matrix. Under what conditions on A is the matrix $A^T A$:

- Symmetric?
- Nonsingular?
- Positive definite? (Recall that a matrix is positive definite if $\langle Av, v \rangle > 0$ for all $v \neq 0$.)

In each case fully justify your answer, proving your statement and/or providing counterexamples as needed.

Exercise 3 (15 pts). Which of the following matrices are orthogonal? $A_1 := \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $A_2 := \begin{bmatrix} 4 & 0 \\ 0 & \frac{1}{4} \end{bmatrix}$, $A_3 := \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$, $A_5 := A_1 A_3$, $A_6 := A_3 A_2$, $A_7 := A_2 A_3 A_2^{-1}$.

Exercise 4 (35pts). Exercise 6.4 in Trefethen and Bau.