Homework 4 - Due Wed. Sep. 23rd
Math 561

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Homework policy as in the previous homework.

Assignment

Exercises 7.4 [10 pts] , 7.5 [10 pts], 9.1 [10 pts] in Trefethen's book.

Besides the above, we will play with Legendre polynomials a bit more. Part of this may be useful to solve 9.1 itself, so you may want to read what follows before you start working on 9.1. [20 pts]

- Create a function `Experiment1` with inputs ν, $MaxDegree$, and outputs $x, A, Q, R, Qc, Rc, Qm, Rm$. Since we are going to vary ν in what follows, we should pay attention to normalization. When ν changes, the dimension of the columns of the Q matrices changes, and since those columns are normalized by Euclidean norm, i.e. $\sum_{i=1}^{2^\nu+1} |Q(i,k)|^2 = 1$, this will cause Q matrices that are not really comparable across different values of ν: look for a suitable normalization factor a_ν that makes them comparable. You will need similar normalizations below.
- `Experiment1` first creates the vector x with $2^\nu + 1$ equispaced points in $[-1, 1]$ (you may use the Matlab function `linspace` for this)
- Then `Experiment1` creates a $(2^\nu + 1) \times MaxDegree$ matrix A whose j-th column is the monomial x^{j-1} evaluated on the grid x you constructed.
- Finally, `Experiment1` computes the reduced QR decompositions of A using `qr`, `clGS` and `mGS` (the last two functions may be downloaded from the webpage), storing the results in $[Q, R]$, $[Qc, Rc]$ and $[Qm, Rm]$ respectively.

Using `Experiment1`

- Call `Experiment1` with $\nu = 15$ and $MaxDegree = 30$, and store the resulting Q in a variable Qtr: we will consider the columns of Qtr as the "true" Legendre polynomials. Why didn't we choose $\nu = 30$? Why is it important to compute $\texttt{qr}(A, 0)$ instead of `qr`(A)?
- Create a function `Experiment1a`, which should do all of the following, given input $MaxDegree$. As a function of $\nu = 5, \ldots, 15$, let Q_ν, Qc_ν, Qm_ν be the Q matrices computed by running `Experiment1` with parameter ν. We want to compare these to Qtr. In order to do this, restrict the columns of Qtr appropriately (by discarding the appropriate rows, corresponding to appropriate points on the x-axis) so that they represent the ("true") Legendre polynomials restricted to the $(2^\nu + 1)$ equispaced points, and let this matrix be Qtr_ν. In the same figure, plot, as a function of ν, $||a_\nu Q_\nu - a_{tr,\nu} Qtr_\nu||_2 / ||a_{tr,\nu} Qtr_\nu||_2$ (here a_ν is a normalization factor, possibly different from the one above, that you have to suitably define) and $\max_i ||a_\nu Q_\nu(:, i) - a_{tr,\nu} Qtr_\nu(:, i)||_2 / ||a_{tr,\nu} Qtr_\nu(:, i)||_2$ (this is Matlab pseudo-code). In a second figure, similar plots (with different markers for different plots (these may be obtained with commands like `plot(x,y,'x')`...)) comparing Qc_ν and Qm_ν to Qtr_ν (all appropriately normalized). All the plots should be in log scale in the vertical direction.
- Call `Experiment1a` with $MaxDegree = 6$. What does the plot with Q_ν and Qtr_ν tell us about the role of ν? [this is related to question (c) of 9.1 in the book] What do the other plots tell us? Same questions for $MaxDegree = 30$.

- Finally, plot $||a_\nu Qc_\nu(:,i) - a_{tr,\nu} Qtr_\nu(:,i)||_2 / ||a_\nu Qc_\nu(:,i)||_2$ and $||a_\nu Qc_\nu(:,i) - a_{tr,\nu} Qtr_\nu(:,i)||_\infty / ||a_\nu Qc_\nu(:,i)||_\infty$ as a function of i. Discuss what you see in the plot. Same for Qm_ν and Q_ν replacing Qc_ν (all suitably normalized).