

Homework 7 - Due Wed. Oct. 21st Math 561

Dr. Mauro Maggioni
Office: 319 Gross Hall
Phone: 660-2825
Web page: www.math.duke.edu/~mauro
E-mail: [mauro.maggioni at duke.edu](mailto:mauro.maggioni@duke.edu)

Homework policy as in the previous homework.

Assignment

Exercise 14.1 (35 pts), 15.1 (30 pts), 16.2 (35 pts)

Possible simple exercises in preparation of midterm

The following exercises are taken directly from, or strongly inspired by, exercises in M. Heath's Scientific Computing textbook, in which you may find a large selection of exercises:

- 1.4 True or false: a good algorithm will produce an accurate solution regardless of the condition of the problem being solved
- 1.2 True or false: using higher-precision arithmetic will make an ill-conditioned problem better conditioned
- 1.7 True or false: if two real numbers are in \mathbb{F} (the set of floating point numbers) then the result of a real arithmetic operation on them will also be representable as a number in \mathbb{F}
- 2.6 True or false: An underdetermined system of linear equations $Ax = b$, where A is $m \times n$ with $m < n$, always has a solution.
- 2.25 True or false: If A is $n \times n$ nonsingular, then $\text{cond}(A) = \text{cond}(A^{-1})$.
- 2.52 In general which matrix norm is easier to compute, $\|A\|_1$ or $\|A\|_2$? Why?
- 2.67 Let A be a square matrix and c a scalar. Which of the following is always true: (a) $\|cA\| = |c| \|A\|$; (b) $\text{cond}(cA) = |c| \text{cond}(A)$.
- 3.3 True or false: At the solution to a linear least squares problem $Ax = b$, the residual vector $r = b - Ax$ is orthogonal to $\text{range}(A)$.
- 3.41 Let A be a $m \times n$ matrix. (a) What is the maximum number of nonzero singular values that A can have? (b) If $\text{rank}(A) = k$, how many nonzero singular values does A have?
- 3.6 (a) What is the Euclidean norm of the minimum residual vector for the following linear least squares problem?

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

- (b) What is the solution vector x for this problem?

3.18 Suppose that you are computing the QR factorization of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \end{bmatrix}$$

by Householder transformations.

- How many Householder transformations are required?
- What does the first column of A become as a result of applying the first Householder transformation?
- What does the first column of A then become as a result of applying the second Householder transformation?