Topics in Statistical Learning, High Dimensional Geometry and Random Matrices
Math 790 - Spring 2013

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Class: Math 790-50.03
WF 10:05-11:20AM
Physics 227
Mar 20 - Apr 17

Synopsis of course content

We will cover some basic materials in random matrix theory (with applications to compressed sensing and signal processing), nonparametric statistical estimation and machine learning, and problems about the geometry of high dimensional data sets.

- **Random matrices**. Basic theory of random matrices, following [8]: basic concentration inequalities, subgaussian random variables, singular values of random matrices. *Applications*: to compressed sensing theory; to numerical linear algebra (a.k.a. how to compute quickly highly accurate low-rank approximate Singular Value Decompositions, with high probability [9]).
- **Nonparametric estimation**: Basic results in nonparametric density estimation and nonparametric regression (e.g. following the first chapter [7]) in low dimensions. Obstructions in the high-dimensional setting, curse of dimensionality. *Applications*: denoising of signals (the classic Donoho-Johnstone paper [4] and the compressed sensing results).
- **Approximation theory**. A primer in nonlinear approximation of functions [3], especially for wavelets and other multiscale approximations. Multiscale approximation of functions in high dimensions [2]. Attacking the curse of dimensionality.
- **Multiscale Analysis in High dimensions**. Multiscale geometric constructions in metric spaces, associated algorithms and applications. Multiscale SVD and Geometric Multiresolution analyses, and their applications to dictionary learning, regression, manifold learning, compressive sensing [6, 1, 5].
- **Optimal transport**. A primer in optimal transport theory and Wasserstein metrics between distributions. Current research: multiscale approximation theory in the space of probability measures with respect to Wasserstein metrics.

References

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[4] D. L. Donoho and I. Johnstone, *Ideal denoising in an orthonormal basis chosen from a library of bases*, tech. rep., Stanford University, 1994.

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