

Homework 1 - Due Wed. Feb. 14th

Introduction to Harmonic Analysis and its Applications

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Homework Policies

Homework is due to the TA on the due date before the beginning of class.

No late homework will be accepted without permission obtained in advance. Johns Hopkins' policies apply with no exceptions to cases of incapacitating short-term illness, or for officially recognized religious holiday. You may, and are encouraged to, discuss issues raised by the class or the homework problems with your fellow students and both offer and receive advice. However all submitted homework must be written up individually without consulting anyone else's written solution.

The submission of homework that require numerical work on a computer should include the following: printout of the code used to solve the problems, of its inputs and of its outputs. The code should be written clearly, copiously commented, and input/outputs of the code clearly documented in format and content. The specific outputs requested by the exercise should be discussed in your writeup as needed in order to answer the questions in the problems. For example if the problem asks you to compare the results of two algorithms for solving a given linear system, you should exhibit the the code for the two algorithms, commented, the input to the algorithms and the two outputs, and comment on whether the results are the same or not, why, etc...

Assignment

Review your real analysis and linear algebra. For real analysis, besides basic calculus, review notions of convergence for sequences and series; as well as for functions and series of functions (pointwise and uniform convergence); for linear algebra, we will soon be using (abstract) vectors spaces, norms, inner products, linear operators, bases.

Also, review the most basic parts of complex analysis, in particular the definition of e^z , $z \in \mathbb{C}$ via power series, Euler's identity $e^{i\theta} = \cos(\theta) + i \sin(\theta)$ for $\theta \in \mathbb{R}$ (what happens if $\theta \in \mathbb{R}$ is replaced by $z \in \mathbb{C}$?), and other relationships between complex exponentials and sine and cosine functions (power series expansions, trigonometric identities, etc...).

Exercises

Exercise 1 (40pts). Exercise 4 from chapter 2 in Stein and Shakarchi's book: consider the 2π -period odd function defined on $[0, \pi]$ by $f(\theta) = \theta(\pi - \theta)$: (a) draw the graph of f , and (b) compute the Fourier coefficients of f , and show that

$$f(\theta) = \frac{8}{\pi} \sum_{k \text{ odd} \geq 1} \frac{\sin k\theta}{k^3}.$$

Exercise 2 (30pts). Prove that if f is an even 2π -period function, i.e. $f(\theta) = f(-\theta)$ for all $\theta \in [-\pi, \pi]$, then the Fourier series can be written as a cosine series.

Exercise 3 (30pts). Prove that the Fejér kernel is given by $F_N(\theta) = \frac{\sin^2(N\theta/2)}{N \sin^2(\theta/2)}$.