Homework 4 - Due Wed. Apr. 11th
Introduction to Harmonic Analysis and its Applications

Dr. Mauro Maggioni
Web page: www.math.jhu.edu/~mauro
Office: 411 Krieger Hall
E-mail: myfirstname.maggioni@jhu.edu

Homework Policies
As in Homework 1.

Assignment

Exercises

Exercise 1 (50pts). Exercise 5.2 in S. Mallat’s book. Prove that for any \( \lambda \in \mathbb{R} \setminus \{0\} \) the system \( \{e^{2\pi ikn/\lambda}\}_{k \in \mathbb{Z}} \) is a tight frame of \( L^2([0,1]) \), and compute the frame bound. [Hint: do no forget to show completeness].

Exercise 2 (50pts). Consider \( d \) vectors \( v_1, \ldots, v_d \) spanning a subspace \( V \) in \( L^2(\mathbb{R}) \) (or \( L^2(S^1) \), or \( L^2([0,1]) \) if you prefer). First show that \( v_1, \ldots, v_d \) is a frame for \( V \). Then construct an example of a system \( \{v_1, \ldots, v_d\} \), for each \( d \), where as \( d \to +\infty \), the frame bounds \( A, B > 0 \) tend to 0 and \( +\infty \) respectively.