Homework 1 - Due Wed. Feb. 13th
Introduction to Harmonic Analysis and its Applications

Dr. Mauro Maggioni **Office**: 405 Krieger Hall
Web page: `https://mauromaggioni.duckdns.org` **E-mail**: firstnamesecondnamejhu@icloud.com

Homework Policies

Homework is due on the due date before the beginning of class.
No late homework will be accepted without permission obtained in advance. Johns Hopkins' policies apply with no exceptions to cases of incapacitating short-term illness, or for officially recognized religious holiday. You may, and are encouraged to, discuss issues raised by the class or the homework problems with your fellow students and both offer and receive advice. However all submitted homework must be written up individually without consulting anyone else's written solution.

The submission of homework that require numerical work on a computer should include the following: printout of the code used to solve the problems, of its inputs and of its outputs. The code should be written clearly, copiously commented, and input/outputs of the code clearly documented in format and content. The specific outputs requested by the exercise should be discussed in your writeup as needed in order to answer the questions in the problems. For example if the problem asks you to compare the results of two algorithms for solving a given linear system, you should exhibit the the code for the two algorithms, commented, the input to the algorithms and the two outputs, and comment on whether the results are the same or not, why, etc...

Exercises marked by (*) are not mandatory, but give you extra credit that will accumulate throughout the course and may affect your final grade.

Assignment

Review your real analysis and linear algebra. For real analysis, besides basic calculus, review notions of convergence for sequences and series of functions, in particular pointwise and uniform convergence (we mentioned them in class but only briefly, we will be using them soon, and while I will be refreshing your memory, I will not go into a detailed review); for linear algebra, we will soon be using (abstract) vectors spaces, norms, inner products, linear operators, bases; Cauchy-Schwartz inequality.
Review basics of complex numbers. Also, review the most basic parts of complex analysis, in particular the definition of e^z, $z \in \mathbb{C}$ via power series, Euler's identity $e^{i\theta} = cos(\theta) + i\sin(\theta)$ for $\theta \in \mathbb{R}$ (what happens if $\theta \in \mathbb{R}$ is replaced by $z \in \mathbb{C}$?), and other relationships between complex exponentials and sine and cosine functions (power series expansions, trigonometric identities, etc...).
Study textbook. Chapter 1 up to Section 1.3.

Exercises

Exercise 1 (100pts). Let K be a function defined on the unit square $Q := [0,1] \times [0,1]$. Define the map $f \mapsto Kf$ defined, for functions defined on $[0,1]$, by $(Kf)(x) = \int_{[0,1]} K(x,y)f(y)dy$, for $x \in [0,1]$, so that Kf is a function on $[0,1]$.

- Assume $K \in \mathcal{C}(Q)$, and consider the map above for $f \in \mathcal{C}([0,1])$. Is the map well-defined? To which space does Kf belong? If yes, does it define a linear operator? If yes, is such a linear operator bounded/continuous/Lipschitz from $\mathcal{C}([0,1], ||\cdot||_\infty)$ onto itself? If yes, can you find an upper bound on the norm of such operator; if no make sure you show why it is not bounded/continuous/Lipschitz. (*) If it is bounded, can you find the exact norm of the operator?

- Same questions as above, but for $K \in L^2(Q)$, $f \in L^2([0,1])$, and the linear operator viewed from $L^2([0,1], ||\cdot||_2)$ onto itself. Give an example of a $K \in L^2(Q)$ that is not in $\mathcal{C}(Q)$.

Hint: use Cauchy-Schwartz inequality in the second part