

**Homework 2 - Due Wed. Feb. 20th**  
**Introduction to Harmonic Analysis and its Applications**

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## Homework Policies

As in homework 1.

Exercises marked by (\*) are not mandatory, but give you extra credit that will accumulate throughout the course and may affect your final grade.

### Exercises

*Exercise 1* (70pts). Exercise 4 from chapter 2 in Stein and Shakarchi's book: consider the  $2\pi$ -period odd function defined on  $[0, \pi]$  by  $f(\theta) = \theta(\pi - \theta)$ : (a) draw the graph of  $f$ , and (b) compute the Fourier coefficients of  $f$ , and show that

$$f(\theta) = \frac{8}{\pi} \sum_{k \text{ odd } \geq 1} \frac{\sin k\theta}{k^3}.$$

Is this function continuous on the circle? continuously differentiable on the circle? in  $L^2$  of the circle?

*Exercise 2* (30pts). Prove that if  $f$  is an even  $2\pi$ -period function, i.e.  $f(\theta) = f(-\theta)$  for all  $\theta \in [-\pi, \pi]$ , then the Fourier series can be written as a cosine series (i.e. if you expand the complex exponentials into cosine and sines, the sine terms vanish).