Homework 4 - due Wed. Feb. 26th
High-Dimensional Approximation, Probability, and Statistical Learning

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Homework Policies

As in the first homework set.

Assignment

Study/review the parts discussed in class of Chapter 4 of R. Vershynin's High-Dimensional Probability lectures notes/book.

Exercises

Exercise 1 (30pts). Exercises 4.4.3.

Exercise 2 (70pts). Experiment numerically with Theorem 4.4.5: write code to construct, for any m, n, a matrix A as in Theorem 4.4.5. Compute $||A||$ and plot it as a function of n, of m (separately, for easily readable plots), and as a function of $m = n = p$ (square matrices of increasing size), and plot both $\mathbb{E}||A||$ and its standard deviation (as a function of m, n), estimated from the samples drawn, over multiple draws of the matrix A, for each fixed n, m. Does Theorem 4.4.5 make accurate predictions? Do m, n need to be really large for the predictions to be reasonably accurate?

Now look also, in the case of $m = n = p$, at all the singular values of $\frac{1}{\sqrt{m}}A$ (not just the largest one, which is $||A||/\sqrt{m}$): they are random, and you can look at their distribution, as a histogram (computed over multiple realizations of A) over a fixed interval (that's why we rescaled by $1/\sqrt{m}$). Do the histograms seem to converge as $m \to +\infty$? to what?